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Investigación

Introduction to Scalar Field Dark Matter Structure Formation

Introducción al Campo Escalar de la Materia Oscura de Formación de Estructura

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Resumen

En este trabajo se introduce un potencial Φ^2 para describir la dinámica de nuestro candidato de campo escalar para ser la materia oscura del Universo, y uno de los principales ingredientes necesarios para obtener la estructura a gran escala que vemos hoy en día. Aquí calculamos y simulamos algunos de estos ingredientes necesarios para obtener de manera cualitativa, el perfil de densidad de materia a grandes escalas que observamos. Comenzamos asumiendo que las semillas del universo temprano eran pequeñas fluctuaciones que viván dentro de un fondo homogéneo de Friedmann-Lemaitre-Robertson-Walker (FLRW), las cuales se estudian utilizando la Teoría Lineal de Perturbaciones, estas fluctuaciones crecieron debido a la inestabilidad gravitacional, así dando lugar a lo que hoy observamos: galaxias, clusters de galaxias, etc. En este trabajo utilizamos la métrica perturbada más general para las fluctuaciones escalares, las cuales usamos para obtener tanto las ecuaciones homogéneas de Einstein como las perturbadas. Al calcular la densidad de energía utilizamos la conservación del tensor de energá-momento, el cual también se encuentra perturbado, y que junto con las ecuaciones de Einstein y la ecuación perturbada de Klein-Gordon nos dan un conjunto completo de ecuaciones a resolver.

***Palabras Claves*:** Materia Oscura, Campos Escalares, Teoría de Perturbaciones, Estructura a Gran Escala.

Abstract

In this work we introduce a Φ^2 potential to describe the dynamics of our scalar field (SF) candidate to be the dark matter in the Universe, and as one of the main ingredients needed to obtain the large-scale structure we see today. We calculate and simulate some of the ingredients needed to obtain the matter density profile of the large-scale structure we observe now a days in a qualitative way. We begin assuming that the seeds of the early universe were small fluctuations that lived inside an homogeneous Friedmann-Lemaitre-Robertson-Walker (FLRW) background, and which are studied using the Linear Perturbation Theory, these fluctuations then grew because of gravitational instability, this way giving birth to what we see today: galaxies, clusters of galaxies, etc. We then work with the most general perturbed metric for scalar perturbations, which we use to obtain the homogeneous and perturbed Einstein's equations together with the help of General Relativity. In calculating the energy density we used the conservation of the energy-momentum tensor, which is now also perturbed, and that together with the Einstein's equations and the perturbed Klein-Gordon equation gave us our complete set of equations to be solved.

***Keywords*:** Dark Matter, Scalar Fields, Perturbation Theory, Large-Scale Structure.

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1. Introduction

The cosmic microwave background radiation (CMB) is one of the main proofs for the homogeneous and isotropic model of the Big Bang. The anisotropies of the CMB are related with small perturbations that are found to be inside a perfectly smooth background, and which are believed to be the seeds on the formation of galaxies and large-scale structure of the Universe. We then assume that in the past these small deviations existed in our homogeneous Universe. For this we propose a cosmological model that includes a scalar field to be the dark matter which can have these initial inhomogenities, than will later evolve to form galactic structure.

Nowadays we know that the Universe is not perfectly homogeneous and isotropic; there are galaxies and clusters of galaxies; whose large-scale distribution is not given randomly. It has been possible to observe that such anisotropies on the temperature of the CMB predict the existence of small deviations of the uniform density of space, seen also at large-scales.

For cosmological models that involve scalar fields, it is usually supposed that the scalar field has (spatial) density fluctuations at cluster scales or below. This is because in the linear perturbation theory, the mass of the field is very small and because of this it does not feel smaller or equal fluctuations to a tenth of a Megaparsec. Not taking into account the perturbations in the mass of the scalar field dark matter results in a good approximation when the perturbations are small. If we take them into account, we have to be careful, because then the local metric of FLRW would not be a good approximation anymore. In this case, both linear and non-linear perturbations might modify the evolution of the dark matter perturbations, these affecting at the same time the evolution of structure formation.

As the Universe evolves, matter is initially accumulated in very dense regions not caring how small the initial density could be; eventually enough matter will be gathered into this region to create the structure.

The most accepted cosmological models for understanding the mechaof structure formation are those that contain dark matter and dark energy, with a possibly negative equation of state as the main constituyents of the Universe, [1].

Both dark matter and dark energy can be described by a dynamical scalar field that rolls in its potential $V(\Phi)$, although there is not yet a general agreement on the correct form that this scalar potential $V(\Phi)$ should have, [2]-[4].

The fundamental questions in this kind of problem are then: What do we mean with non-uniform? and how could we quantify the distribution of the galaxies that are now being observed? We begin then by understanding the concept of uniformity which will directly lead us to the answer of the second question. For this, imagine that galaxies are distributed in some random way in the Universe, and we want to understand if the mechanism responsible of this distribution operates in a uniform way. Such process has to give the same probability of having a galaxy in each position of space.

The aim is to reconstruct the observed Universe, assuming that at some stage in the past the where small deviations in the homogeneity of space. It is believed that cosmic structure could have had its origins due to these deviations that where amplified by the gravitational instability of small fluctuations of the early Universe.

While these homogeneities are kept small, their growth can be studied with linear perturbation theory. We should emphasis that upon assuming the existence of these small inhomogeneities at some initial stage, the cosmological model proposed should reproduce and satisfy such initial conditions.

We will introduce a scalar field Φ as a candidate to be the dark matter both at cosmological and galactic scales in a flat Universe with 97% of the matter unknown but of great importance at the cosmological level, where we have to take into account the possibility that the structures could be made up of this kind of matter, i. e., dark matter may contribute to the formation of structure.

The main objective of this work is to introduce a scalar field dark matter model and assume that this dark matter is a real scalar field which involves an auto-interacting potential $V(\Phi) = \frac{1}{2}m^2\Phi^2$, where the mass of the SF Φ is defined as $m_\Phi = \ddot{V}_\Phi = 0$ and the dark matter is affected by radiation only indirectly, through the gravitational potential.

Then, to treat these anisotropies we need to know how the perturbations that act upon the dark matter evolve. This will give us energy density needed to obtain the right amount of mass in galaxies.

We develop the theory and the simulations needed to obtain the right spectra, based on the facts mentioned before, and referring to the non-perturbed model (without fluctuations) as the background, from where the small fluctuations will be evolved.

2. Theory

The high degree of isotropy observed in the CMB, together with the Copernican principle, give us the grounds to believe that the large-scale structure in the observable Universe can be correctly described by the FLRW model, which is isotropic and spatially homogeneous. Nowadays it is believed that the exact deviations to the FLRW model are small enough that can be described considering the linear perturbation theory. The model of introducing perturbations to the FLRW metric was introduced by Lifshitz (1946). In this theory we distinguish between the space-time of a background, FLRW standard model, and a perturbed space-time, which represents the physical Universe. In this section we present the fundamental equations that are needed for the analysis of the perturbations of the scalar field in a FLRW space-time, here we have taken $c \equiv 1$, with c the speed of light. After introducing the perturbed metric tensor in a FLRW background we mainly consider scalar perturbations. We then give the equations for the conservation of energy and momentum, and Einstein's field equations for a FLRW perturbed metric.

We consider first order perturbations over a FLRW background, such that the metric tensor can be separated as

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g_{\mu\nu},$$

where the background metric $g^{(0)}_{\mu\nu}$ is given by

$$g^{(0)}_{\mu\nu} = a^2(-1, \delta_{ij}),$$

where δ_{ij} is the 3-dimensional metric with constant curvature. It is considered that the background is the non-perturbed space-time, that is described by the FLRW metric. The metric tensor has 10 independent components in 4 dimensions. For linear perturbations it is better to separate the metric perturbations in different parts called scalars, vectorial and tensorial depending in their properties. The reason to separate the metric perturbations is because they themselves decouple in the perturbed linear equations and so it is easier to distinguish them.

The scalar perturbations can always be constructed by a scalar quantity, its derivatives or any background quantity as the metric δ_{ij}. We can then have a perturbed scalar metric to first order in terms of four scalars ψ (lapse function), ϕ (gravitational potential), B (shift) and E (anisotropic potential) , where

$$\begin{aligned} \delta g_{00} &= -a(\eta)^2 2\psi, \\ \delta g_{0i} &= \delta g_{i0} = a(\eta)^2 B_{,i} \\ \delta g_{ij} &= -2a(\eta)^2(\phi\delta_{ij} - E_{,ij}). \end{aligned} \tag{1}$$

with η the conformal time.

From this, we get the most general perturbed line element

$$\begin{aligned} ds^2 = a(\eta)^2[&-(1+2\psi)d\eta^2 + 2B_{,i}\, d\eta dx^i \\ &+ [(1-2\phi)\delta_{ij} + 2E_{,ij}]dx^i dx^j], \end{aligned} \tag{2}$$

where δ_{ij} is the background metric.

By definition any perturbation done in any quantity its the difference between its value in some event in real space-time, and its corresponding value, through the metric, in a event in the background.

The energy-momentum tensor for a scalar field minimally coupled to the gravitational potential is given by

$$T^\nu_\mu = \Phi_{,\mu}\Phi_{,\nu} - \left[\frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\Phi_{,\alpha}\Phi_{,\beta} + g_{\mu\nu}V(\Phi)\right]. \tag{3}$$

We separate the tensorial quantities in a background value and a perturbation according to

$$\boldsymbol{T} = \boldsymbol{T}_0 + \delta\boldsymbol{T}, \tag{4}$$

where the value for the background is only time dependent $\boldsymbol{T}_0 \equiv \boldsymbol{T}_0(\eta)$, while the perturbations depend on both time and space coordinates $x^\mu = [\eta, x^i]$, i. e, $\delta\boldsymbol{T} \equiv \delta\boldsymbol{T}(x^\mu)$.

The sign convention is taken to be $(+++)$. When considering perturbations on the metric, we are taking the scalar field theory in a homogeneous and isotropic space-time.

The scalar field Φ is also separated in a background term and a first order perturbation according to (4),

$$\Phi(x^\mu) = \Phi_0(\eta) + \delta\Phi(x^\mu),$$

the potential $V \equiv V(\Phi)$ is separated in an analogous way where in this case,

$$\delta V = V_{,\Phi}\,\delta\Phi,$$

and $V_{,\Phi} \equiv \frac{\partial V}{\partial \Phi}$.

The energy-momentum tensor for the scalar field with potential $V(\Phi)$ is also separated in a background term and a fist order perturbation. For the zero components of (3) we have:

$$T^0_0 = -\hat{\rho} = -\left(\frac{1}{2}\dot{\Phi}^2_0 + V_0\right),$$

$$T^i_j = p = \left(\frac{1}{2}\dot{\Phi}^2_0 - V_0\right)\delta^i_j.$$

And for the zero order components of Einstein's tensor;

$$G^0_0 = -3\frac{\dot{a}^2}{a^2},\ G^i_j = -\left(\frac{\dot{a}^2}{a^2} + 2\frac{\ddot{a}}{a}\right).$$

Using $G^i_j = 8\pi G T^i_j$, for the equations of motion for an homogeneous background with scale factor $a(t)$ and Hubble parameter $H \equiv \frac{\dot{a}}{a}$ we have:

$$8\pi G\rho = 3H^2, \tag{5}$$

$$4\pi G(\rho + 3p) = -3\dot{H}. \qquad (6)$$

Equations (5) and (6) are usually known as Friedmann equations.

From these set of equations we can notice that when talking about the background, the only zero order quantities are the energy density ρ, the pressure p and the volumetric expansion $3H$.

Scalar fields are governed by Klein-Gordon's equation, that can be obtained through the conservation equation

$$\nabla_\mu T^{\mu\nu} = 0,$$

and is given by

$$\ddot{\Phi}_0 + 3H\dot{\Phi}_0 + V_{,\Phi} = 0.$$

Remember that the perturbed metric has 10 independent components inside a four dimensional space, but we only have six independent Einstein equations. We then have to fix the remaining four degrees of freedom by making a choice of the gauge.

In this section we develop the theory through, the Newtonian gauge, it is the most convenient gauge for the study of scalar perturbations.

As alway we are still considering a spatially flat background with scalar perturbations on the metric. Remember that in this case we have set the speed of light c equal to unity.

We now have the basic equations to study the non-perturbed FLRW space, which is being occupied by minimally coupled real scalar field.

We can estimate the dimensions for the background and perturbed variables, in the unit system where $c \equiv 1$. As $8\pi G = [M^{-1}T^2]$, where M and T indicate mass dimensions (energy) and time (length) respectively, we then have:

$$[\rho] = [p] = [\delta\rho] = [\delta p] = [MT^4],$$
$$[\dot{\Phi}_0\delta\phi] = [MT^3], [\phi] = [\psi] = 0,$$
$$[\Phi_0] = [\delta\Phi] = [M^{1/2}T^{-1}],\ [V] = [MT^4].$$

We now derive the perturbed evolution equations for the different quantities mentioned above; the scalar perturbation $\delta\Phi$ and the scalar potential ϕ. For the perturbed energy-momentum tensor, we have:

$$\begin{aligned}
\delta T^0_0 &= -\delta\rho_\Phi = -(\dot{\Phi}_0\dot{\delta\Phi} - \dot{\Phi}_0^2\psi + V_{,\Phi}\,\delta\Phi),\\
\delta T^0_i &= -\frac{1}{a}(\dot{\Phi}_0\delta\Phi_{,i}\,),\\
\delta T^i_j &= \delta P_\Phi = (\dot{\Phi}_0\dot{\delta\Phi} - \dot{\Phi}_0^2\psi - V_{,\Phi}\,\delta\Phi)\delta^i_j. \qquad (7)
\end{aligned}$$

In the above equations the dots denote differentiating with respect to the cosmological time t, which is related to the conformal time by $d/d\eta = a(d/dt)$.

Using the energy-momentum tensor and the equation for the conservation of energy we obtain Klein-Gordon's equation or the equation for the scalar field to first order

$$\nabla_\mu T^\mu_\nu = T^\mu_{\nu,\mu} + \Gamma^\mu_{\alpha\mu}T^\alpha_\nu - \Gamma^\alpha_{\nu\mu}T^\mu_\alpha = 0,$$

where the connection coefficients (Christoffel symbols) are given by

$$\Gamma^\gamma_{\beta\mu} = \frac{g^{\alpha\gamma}}{2}(g_{\alpha\beta,\mu} + g_{\alpha\mu,\beta} - g_{\beta\mu,\alpha}).$$

Then, using the perturbed metric and the perturbed energy-momentum tensor to first order we obtain the equation for the field to first order

$$\begin{aligned}
&\ddot{\delta\Phi} + 3H\dot{\delta\Phi} - \frac{1}{a^2}\nabla^2\delta\Phi + V_{,\Phi\Phi}\,\delta\Phi + 2V_{,\Phi}\,\phi\\
&\quad - \dot{\Phi}_0\dot{\phi} - 3\dot{\Phi}_0\dot{\psi} - \frac{\dot{\Phi}_0}{a}\nabla^2[a\dot{E} - B] = 0. \qquad (8)
\end{aligned}$$

The Newtonian gauge is defined by making $B = E = 0$, i. e., the shift and anisotropic potential are taken as zero, [5].

Note that the Newtonian gauge (also called longitudinal gauge) is a gauge with restrictions, this gauge is only applied to the scalar modes of the perturbations in the metric; the vector and tensor degrees of freedom are eliminated since the beginning. In this case only the scalar perturbations are taken into account.

One of the advantages of working with this gauge is that the metric tensor $g_{\mu\nu}$ becomes diagonal, and this allows for the maths to become easier. Another advantage is that ϕ will now play the part of the gravitational potential making easier for us to give a physical interpretation of the results. In this case, the two scalar potentials ϕ and ψ become identical $\psi - \phi = 0$. Usually this equation contains a term of anisotropic stress, which vanishes in the case of a scalar field. Altogether, to first order, the perturbed Einstein's equations $\delta G^i_j = \kappa^2\delta T^i_j$ for a scalar field in the Newtonian gauge are

$$\begin{aligned}
-8\pi G\delta\rho_\Phi &= 6H(\dot{\phi} + H\phi) - \frac{2}{a^2}\nabla^2\phi,\\
8\pi G\dot{\Phi}_0\delta\Phi_{,i} &= 2(\dot{\phi} + H\phi)_{,i}\,,\\
8\pi G\delta P_\Phi &= 2[\ddot{\phi} + 3H\dot{\phi} + (2\dot{H} + H^2)\phi], \qquad (9)
\end{aligned}$$

which are in accordance with previous results obtained by [6,7] and others.

Equations in (9) can be rearranged to find an equation for ϕ:

$$\begin{aligned}
&\ddot{\phi} + 6H\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + (2\dot{H} + 4H^2)\phi\\
&\quad + 8\pi GV_{,\Phi}\,\delta\Phi = 0. \qquad (10)
\end{aligned}$$

For the perturbed Klein-Gordon equation in the newtonian gauge

$$\delta\ddot{\Phi} + 3H\delta\dot{\Phi} - \frac{1}{a^2}\nabla^2\delta\Phi + V_{,\Phi\Phi}\,\delta\Phi + 2V_{,\Phi}\,\phi - 4\dot{\Phi}_0\dot{\phi} = .0 \quad (11)$$

To solve the evolution equations it has become common to work on Fourier space instead of real space. The beauty of this expansion relays on the fact that each Fourier mode will the evolve independently.

To first order the change to Fourier components are nearly made implicitly. The perturbation $\delta\Phi$ related to its Fourier component $\delta\Phi_k$ by

$$\begin{aligned}\delta\Phi(t,x^i) &= \int d^3k\delta\Phi(t,k^i)exp(ik_ix^i)\\ &= \int d^3k\delta\Phi_k exp(ik_ix^i),\end{aligned} \quad (12)$$

where k^i is the wavenumber, so that the relation $k = 2\pi/\lambda$ is full filed, being λ the wavelength of the perturbations.

The procedure to first order is very simple: the only change in the equations is to replace $\delta\Phi$ by $\delta\Phi_k(k^i)$ and the laplacian by

$$\nabla^2 \to -k^2$$

where $k^2 \equiv k^ik_i$, so that for the gradient operator we have

$$\partial_i \to -ik_i.$$

We now have all the tools needs to rewrite the first order evolution equations in terms of their Fourier components.

The equations for the perturbed scalar field in Fourier space are then:

$$\begin{aligned}8\pi G(3H\dot{\Phi}_0\delta\Phi_k) + \frac{2k^2}{a^2}\phi &= -8\pi G(\dot{\Phi}_0\delta\dot{\Phi}_k - \phi\dot{\Phi}_0^2 + V_{,\Phi}\,\delta\Phi_k),\\ 2(H\phi + \dot{\phi}) &= 8\pi G\dot{\Phi}_0\delta\Phi_k,\\ 2[\ddot{\phi} + 3H\dot{\phi} + (2\dot{H} + H^2)\phi] &= 8\pi G(\dot{\Phi}_0\delta\dot{\Phi}_k - \phi\dot{\Phi}_0^2 - V_{,\Phi}\,\delta\Phi_k).\end{aligned} \quad (13)$$

For the corresponding Fourier transform of equation (10) we have:

$$\ddot{\phi}_k + 6H\dot{\phi}_k + (\frac{k^2}{a^2} + 2\dot{H} + 4H^2)\phi_k + 8\pi G V_{,\Phi}\,\delta\Phi_k = 0. \quad (14)$$

and the Klein-Gordon equation (11) transforms into

$$\delta\ddot{\Phi}_k + 3H\delta\dot{\Phi}_k + (\frac{k^2}{a^2} + V_{,\Phi\Phi}\,)\delta\Phi_k + 2\phi V_{,\Phi} - 4\dot{\phi}\dot{\Phi}_0 = 0. \quad (15)$$

These equations describe the evolution of the perturbations, (13) refers to the energy density ingredient, (14) to the gravitational potential and finally (15) refers to the perturbations of our scalar field.

3. Results

One of the main objectives of this work was to study the qualitative properties of our cosmological model using the theory techniques of a dynamical system. As we have seen before we suppose the Universe governed by the General Theory of Relativity.

Now will study the cosmological evolution of the growth of the scalar field over densities, $\delta\rho_\Phi$, in the linear regime. In order to obtain a numerical solution for the density contrast $\delta = \delta\rho_\Phi/\rho_{\Phi_0}$, the following dimensionless variables are defined

$$\begin{aligned}l_1 \equiv \phi, l_2 \equiv \dot{\phi}/H, y_1 = \delta,\\ z_1 \equiv \frac{\kappa}{\sqrt{6}}\delta\Phi_k, z_2 \equiv \frac{\kappa}{\sqrt{6}}\frac{\delta\dot{\Phi}_k}{H}.\end{aligned} \quad (16)$$

Using these definitions, the evolution equations are transformed into an autonomous system with respect to n, with $n = \ln a$, i. e., $\frac{d}{dt} = H\frac{d}{dn}$, where from now on the upper comma will denote derivative with respect to the e-folding n.

$$
\begin{aligned}
l_1' &= l_2, \\
l_2' &= 3l_2\left(\frac{\Pi}{2}-2\right)+l_1(3\Pi-4)-6z_1us-\frac{k^2s^2l_1}{m^2a^2}, \\
z_1' &= z_2, \\
z_2' &= 3z_2\left(\frac{\Pi}{2}-1\right)-z_1s^2\left(\frac{k^2}{a^2m^2}+1\right)-2usl_1+4l_2x, \\
y_1' &= -3\left[\left(\frac{xz_2-x^2l_1-usz_1}{xz_2-x^2l_1+usz_1}\right)-w_{\Phi_0}\right]y_1+3l_2F_\Phi-\frac{G_\Phi}{H},
\end{aligned}
\tag{17}
$$

where $G_\Phi/H = 2k^2s^2(l_1+l_2)/3a^2m^2\Omega_{\Phi_0}$ and the functions s, x, u and Π are taken from [8] by consistency.

The density contrast parameter is used in cosmology when talking about structure formation to indicate that there exists local agglomeration of matter density.

It is believed that after inflation although the Universe was almost uniform, some regions where more dense than others with very big density contrasts. As the Universe expanded the gravitationally connected masses increased until they began to collapse, allowing the formation of galaxies, clusters, superclusters, etc....

It is common to define the density contrast as $\delta = \delta\rho/\rho$. So making use of our dimensionless variables for the density contrast we have

$$
y_1 = \frac{2[x(z_2-xl_1)+usz_1]}{\Omega_{\Phi_0}}. \tag{18}
$$

We can se that the unknown variables in our system are then ϕ, $\delta\Phi$ and δ. Once these are known we can determine the solution to our dynamic equations.

Next we will integrate our dynamical system, and we will analyze if it can reproduce the observed Universe.

The evolution equations where solved taking the values for the background from [8]. So, taking into account the evolution of the density parameters for the background, we can now compute the evolution of both ϕ which perturbs the metric and the perturbation on the scalar field, which appears in Klein-Gordon's equation.

We start with a perturbation with wavelength $\lambda_k =$ 2Mpc and density contrast $\delta = 1\times 10^{-7}$.

As was mentioned in [8], it is expected that this kind of scalar field forms a BEC, this gives an upper limit for the mass m_Φ of the scalar field, $m_\Phi < 10^{-17}$eV. In our case we took an estimate of approximately 1×10^{-23}eV.

Taking into account all the considerations made above for the initial conditions, we can now completely integrate our dynamical system to obtain the cosmological evolution of ϕ and $\delta\Phi$, which are shown in Fig.1 and Fig.2 respectively,

Fig.2 shows the cosmological evolution of the perturbed scalar field as a function of the scale factor a. As we said before at very early epochs of the Universe the SF was in thermal equilibrium with its surroundings and the temperature of the Universe was very high, dominated mainly by radiation, thus making the amplitude of the fluctuations of the SF very large due to its interactions. As the temperature decreases the SF decouples from the rest of the matter, so that the surrounding interactions are negligible after the decoupling. The evolution of the gravitational potential for this perturbation is shown in Fig.1. Note that the gravitational potential remains constant from $a \sim 10^{-5}$ all along up to the matter-dominated regime.

With these ingredients we can now analyze one of the main quantities when talking about structure formation. We analyze the density contrast in detail, whose expressions are given by (18) or (17). As was mentioned before, this quantity depends on both, the gravitational potential ϕ and the perturbations on the SF, $\delta\Phi$, related to the variables l_1 and z_1 respectively.

We shall remember that our equations are in Fourier space and as a result we obtained a set of independent differential equations for each Fourier mode.

Note: we have normalized the scale factor such that $a = 1$ today, this makes its relation with the redshift z to go as $a = (a+z)^{-1}$, with this comoving and physical coordinates match today.

In Fig.3 we can see that with the imposed initial conditions there is a quick growth in the scale of the density contrast. Recent observations have taken un to think that at very early epochs in the origin of the Universe, there already existed well formed large-scale structure, corresponding to $z \leq 10$, [9].

It is clear from Fig.3 that by $z \leq 10$ there already existed well defined perturbations on the energy density, which could help for the early formation of structure. Then, if clusters could form as early at these z this would imply that the Φ^2 SFDM model could give an explanation for the characteristic masses that are being observed, for more details on the dynamical system see [10].

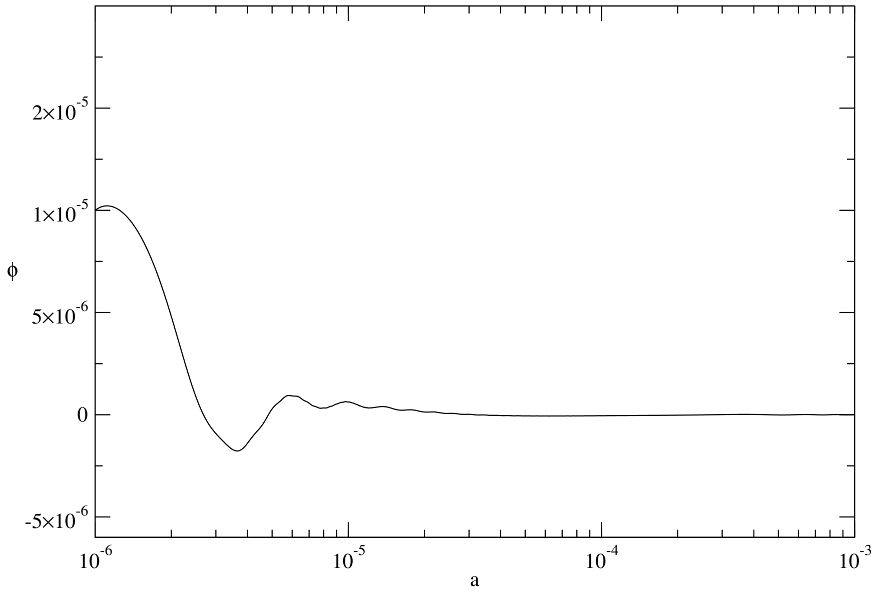

Fig. 1. Evolution of the gravitational potential.

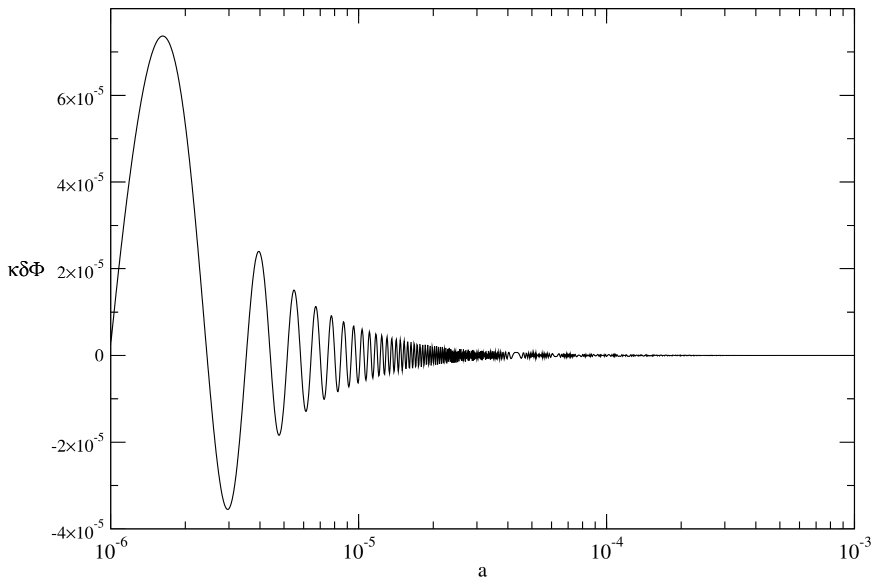

Fig. 2. Evolution of the perturbations on the scalar field.

4. Conclusions

We have investigated the different perturbations that act upon our scalar field with a Φ^2 potential and who seem to have important consequences in the formation of structure in our Universe. We observe that our results do behave as expected depending on the initial conditions.

In this work we supposed a flat Universe, initially homogeneous and built up by a scalar field with potential $\frac{1}{2}m^2\Phi^2$ as dark matter y which upon acts a gravitational potential ϕ for the analysis of first order perturbations.

Once calculated the perturbations over ϕ and $\delta\Phi$, we obtained the density contrast δ, for which we simulated its initial perturbations from $a = 1 \times 10^{-6}$, i. e., before recombination. So, if there already existed perturbations by this epoch we expect to have well formed massive structures at high redshifts, around $z < 10$.

If observations find that big structures already existed and where well formed at this z's, we could

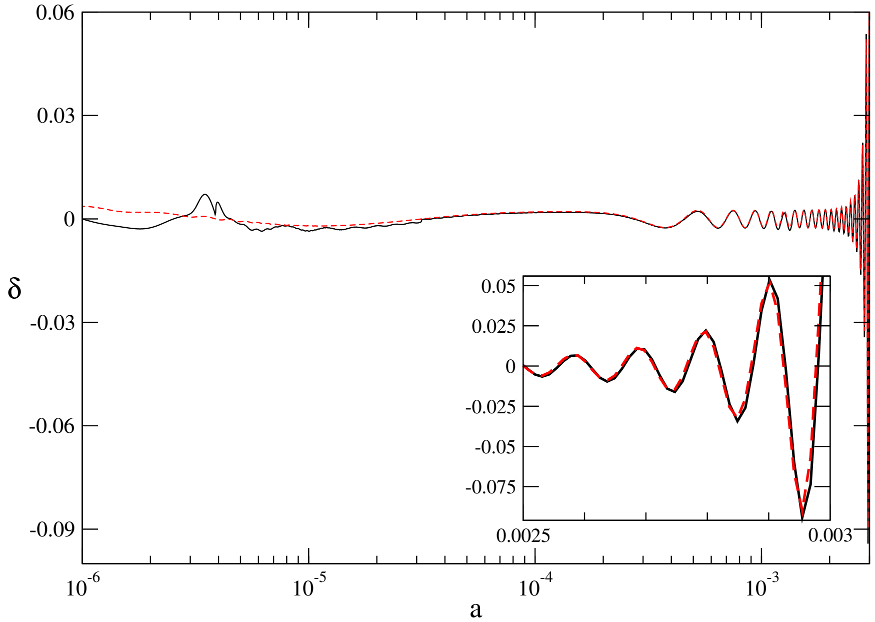

Fig. 3. Evolution of the density contrast δ for $\lambda = 2\text{Mpc}$

have evidence for our cosmological model to be right, predicting such events.

The calculations done in this work are only to first order, so these are valid only in the linear regime and should be taken with care.

If the observations reveal that there is no structure at time dimensions mentioned here, i. e., there is no structure formation before recombination, then our cosmological model can be ruled out because of inconsistency with such observations.

In this work we have seen that the cosmological limits depend upon the model one is working with, therefore it is based on a specific theoretical model, even though it may be in agreement with actual observations, we need much more ingredients to explain mysteries and inconsistencies such as dark matter.

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