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# On The Quality of Drinking Water as a Topic of Multicriterial Decision

## Sobre la Calidad del Agua Potable Como un Tema de Decisión Multicriterio

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**Abstract**

The aim of the present paper is to approach the matter of water quality by resorting to the method of multicriterial mathematical programming. MicrosoftExcel enables the simulation of a mathematical model. The case study presents the simulation of a multicriterial analysis of water quality in Sibiu county, Romania. This particular study shows the means of obtaining more information about water quality, subsequent to the analysis of its characteristics resorting to statistical analysis-specific software, e.g. SPSS 16 software. Given the increase of drinking water worldwide by tens of percentage points in cca two decades, the authors consider that an approach to the quality management of drinking water should represent a key priority of society.

***Keywords*****:** Quality of drinking water, multicriterial modeling, simulation.

**Resumen**

El objetivo de este trabajo es la realización de un estudio del importante asunto de la calidad del agua potable, recurriendo al método de programación matemática multicriterio. MicrosoftExcel permite la simulación de un modelo matemático. El caso bajo estudio presenta la simulación de un análisis multicriterio de la calidad del agua en el condado de Sibiu, Rumania. Este estudio particular muestra los medios de obtención de más información sobre la calidad del agua, subsecuente al análisis de sus caraterísticas recurriendo a software específico de análisis estadístico, por ejemplo, SPSS 16 software. Dado el incremento del subsidio costo capital, de agua potable en todo el mundo, en decenas de puntos porcentuales durante las dos últimas décadas, los autores consideran que para la sociedad es una prioridad clave un estudio de gestión de calidad del agua potable.

***Palabras Claves*****:** Calidad del agua potable, modelando multicriterio, simulación.

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## 1. Introduction

Contemporary society has evinced, at national and international level, a consistent preoccupation for the continuous quality improvement of drinking water [2-4,6,7]. Certain West-European countries are monitoring 45 indicators regarding water quality, whereas the European Economic Commission regulations, approved in 1980, recommend a constant monitoring of 62 characteristics of drinking water. There is a constant preoccupation in all developed countries for controlling water pollution, since the quality of drinking water contributes significantly to the health of a nation. There are special law, in our country as well, meant to fight against water pollution. On the other hand, in certain countries the current consumption of drinking water per capita is very high, therefore specialists recommend a more rational use of water as well as warning about the danger of water resource depletion and pollution in the future. It si estimated that the demand of drinking water shall increase by tens of percentage points in approximately two decades and thus billions of people might suffer from thirst or live in precarious conditions, therefore the interest in the quality management of drinking water is fully justified and should represent a key priority of societies.

The present paper aims to approach the topic of drinking water as a matter of multicriterial decision, considering that multicriterial mathematical modeling enables numberless applications to management and decision theory. According to specialized literature [5], the decision-making process in the field of drinking water quality represents a set of activities that rely on the awareness of the multitude possibilities to act at any given time, analysis of their consequences in relation to a specific goal, selection and implementation of the axiologically optimal action. In this respect, one may resort to the *Multi-Attribute Decision-Making* (MADM) able to solve the *Optimal Choice Problem* (OCP).

Multicriterial programming represents a significant chapter of mathematical optimization, and implicitly of operational research; its significance in decision-problem solving is increasing since the specific methods for this branch of mathematical programming is applicable to a wide array of practical problems, incluing those related to quality management.

Should there be more objective functions, the optimal solution for a function may not be optimal for other functions, hence we introduce the notion of solution achieving “the best compromise” known as non-dominant solution, effective solution, PARET-defined optimal solution, etc. The multicriterial problem or multi-function objective nowadays represents a self-standing chapter of the multiple criteria decision theory.

## 2. Mathematical Multicriterial Modeling of The Quality of Drinking Water

In view of taking into account as many decision criteria as possible, we have designed and developed a specific application for the multi-criteria decision situation in the field of classifying various types of drinking water, selected from various sources, according to the quality properties of water. In order to solve the multicriterial problem, we have started from the decision matrix available in specializedliterature in keeping with the model in Table 1:

where: $V_i = i$ alternative , for $i = 1, 2, 3......m$; $S_h$ objective situation h, for $h = 1, 2, 3......s$; $C_j$ - $j$ criterion for $j = 1, 2, 3......n$; $k_j$ - significance criterion ($j$ criterion weight) for $j = 1, 2, 3...n$; $a_{ijh}$ - i alternative consequence (performance) for the j criterion under $h$ objective circumstances (under the provision of the h situation).

There may be:

(i) Let us mention potential situations:
- $S_1$ = using relevant materials for performing water quality analysis;
- $S_2$ employing qualified staff, experienced for data analysis and interpretation;
- $S_3$ failure of raising funds for performing analysis.
- The problem of multi-criterial decision shall be solved is the above-mentioned situations $(S_1, S_2, S_3)$ have the following occurrence probabilities: $p(S_1) = 0,4, p(S_2) = 0,5, p(S_3) = 0,1$.

(ii) The criteria related to the types of water are:
- $C_1$= pH;
- $C_2$ cost;
- $C_3$ chlorine concentration;
- $C_4$ calcium concentration;
- $C_5$ magnesium concentration.

(iii) The significance criteria coefficients are: $k_1 = 0,3, k_2 = 0,2, k_3 = 0,2, k_4 = 0,15, k_5 = 0,15$.

(iv) Types of water may be:
- $V_1$= Avrig tap water(Sibiu area,Romania);
- $V_2$ Sibiu tap water (Calea Dumbravii from Sibu area, Romania);
- $V_3$ bottled water DORNA(still water, Romania);
- $V_4$ spring water (Sadu area, from Sibiu, Romania);

| **Situations** | $S_1$ | | | | | $S_h$ | | | | | $S_s$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Criterions of decision** | $C_1$ | ... | $C_j$ | ... | $C_n$ | $C_1$ | ... | $C_j$ | ... | $C_n$ | $C_1$ | ... | $C_j$ | ... | $C_n$ |
| *Coefficients of importance* | $k_1$ | ... | $k_j$ | ... | $k_n$ | $k_1$ | ... | $k_j$ | ... | $k_n$ | $k_1$ | ... | $k_j$ | ... | $k_n$ |
| $V_1$ | $a_{111}$ | ... | $a_{1j1}$ | ... | $a_{1n1}$ | $a_{11h}$ | ... | $a_{1jh}$ | ... | $a_{1nh}$ | $a_{11s}$ | ... | $a_{1js}$ | ... | $a_{1ns}$ |
| $V_2$ | $a_{211}$ | ... | $a_{2j1}$ | ... | $a_{2n1}$ | $a_{21h}$ | ... | $a_{2jh}$ | ... | $a_{2nh}$ | $a_{21s}$ | ... | $a_{2js}$ | ... | $a_{2ns}$ |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| $V_i$ | $a_{i11}$ | ... | $a_{ij1}$ | ... | $a_{in1}$ | $a_{i1h}$ | ... | $a_{ijh}$ | ... | $a_{inh}$ | $a_{i1s}$ | ... | $a_{ijs}$ | ... | $a_{ins}$ |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |
| $V_m$ | $a_{m11}$ | ... | $a_{mj1}$ | ... | $a_{mn1}$ | $a_{m1h}$ | ... | $a_{mjh}$ | ... | $a_{mnh}$ | $a_{m1s}$ | ... | $a_{mjs}$ | ... | $a_{mns}$ |

Table. 1. Decision matrix for a multi-criterial problem

- $V_5$ bottled water QLARIVIA (immaculate water, Romania);
- $V_6$ bottled water BORSEC (still water, Romania);
- $V_7$ Cisnădie tap water (Sibiu area, Romania).

The $a_{ijh}$ elements inside the matrices in table 1 represent a product among the values assigned to the $C_j$ criterion on a Likert scale from 1 to 5, i.e. $N_i(C_j)$ of the $kj$ significance coefficient and the $p(S_h)$ situation occurrence probabilities. This product is calculated by the formula:

$$a_{ijh} = N_i(C_j)k_j p(S_h)) \tag{1}$$

where $i = 1, 2, 3, 4, 5, j = 1, 2, 3, h = 1, 2, 3$.

This is a subjective estimation of values and relies on the prior experience of those involved in the process of water sample analysis.

According to formula 1, all $a_{ijh}$ elements are automatically calculated and displayed in Fig. 2:

Based on the decision matrices in Tables of Fig 1 and 2, several decision methods and criteria may be applied, such as: the "mathematical hope" method (whenever the $S_i$ occurrence probabilities are known) which we have chosen to use in the present paper.

The application of the aforementioned method requires that all criteria should be assessed by the same measurement unit on the Likert scale. Therefore:

- First, let us proceed to transforming all consequences in significances, according to the mathematical model:

$$U_{ijh} = (a_{ijh} - a^o_{jh})/a'_{ijh} - a^o_{jh}) \tag{2}$$

where: $U_{ijh}$ - relevance of the $i$ version consequences, for the $j$ criterion given the $h(a_{ijh})$ objective circumstances (situation);

$a'_j$ - the most favourable consequence for the $j$ criterion given the $h$ objective circumstances;

$a^o_j$ - the least favourable consequence for the same $j$ criterion given the $h$ objective circumstances:

- For each $i$ decision alternative and for each status of the $h$ objective circumstances, we have calculated the synthetic relevance (multiplying by the relevance coefficient for each criterion):

$$US_{ih} = \sum K_j u_{ijk} \tag{3}$$

- Based on the synthetic relevance, we ahve drawn up a new matrix including the decision variable on the rows and the potential objective situations on the columns. Thus changed, the problem may be aproached like any other unicriterial decision problem. Taking into account the occurrence probability for the objective circumstances ($S_i$ situations), one may choose the decision option with the highest "mathematical hope":

$$V_{opt} = \sum uS_{ih} p_h \tag{4}$$

where $ph = p(S_h)$ represents the occurrence possibility for the $h$ objective situation:

To conclude, one may notice in the final column the following hierarchy of types of water (alternatives):

(i) $V_5$ bottled water - QLARIVIA (immaculate water) = optimum version.
(ii) $V_6$ drinking water - BORSEC (still water).
(iii) $V_4$ spring water (Sadu area, Sibiu county - Romania).
(iv) $V_3$ bottled water - DORNA (still water).
(v) $V_2$ tap water from Sibiu, Sibiu county - Romania (area - Calea Dumbrăvii street).
(vi) $V_7$ tap water from Cisnădie, Sibiu county - Romania.
(vii) $V_1$ tap water from Avrig, Sibiu county - Romania.

## 3. Simulating the Multicriterial Decision Problem in the Hierarchy of Types of Water, by Means of Microsoft Excel

The result is that the $V_5$ version is optimum, thus the QLARIVIA bottled water is the highest quality drinking water, considering that the managerial expertise of specialists in water analysis played an important part in the selection of $a_{ijh}$ initial elements:

$1)V_5 \;\; 2)V_6 \;\; 3)V_4 \;\; 4)V_3 \;\; 5)V_2 \;\; 4)V_7 \;\; 5)V_1$

| Situations | $p(S_1)=0,4$ | | | | | $p(S_2)=0,5$ | | | | | $p(S_3)=0,1$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Criterions of decision** | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
| *Coefficients of importance* | *0,3* | *0,2* | *0,2* | *0,15* | *0,15* | *0,3* | *0,2* | *0,2* | *0,15* | *0,15* | *0,3* | *0,2* | *0,2* | *0,15* | *0,15* |
| $V_1$ | 4 | 5 | 3 | 3 | 4 | 4 | 5 | 3 | 1 | 3 | 4 | 5 | 3 | 1 | 2 |
| $V_2$ | 3 | 3 | 5 | 4 | 5 | 3 | 3 | 5 | 2 | 4 | 3 | 3 | 5 | 2 | 1 |
| $V_3$ | 5 | 4 | 3 | 5 | 2 | 3 | 4 | 3 | 3 | 5 | 3 | 4 | 3 | 4 | 3 |
| $V_4$ | 4 | 4 | 4 | 2 | 1 | 4 | 3 | 4 | 4 | 2 | 4 | 2 | 4 | 3 | 4 |
| $V_5$ | 5 | 3 | 5 | 1 | 2 | 5 | 3 | 5 | 5 | 1 | 5 | 3 | 5 | 4 | 5 |
| $V_6$ | 2 | 3 | 4 | 3 | 3 | 3 | 2 | 4 | 4 | 2 | 4 | 2 | 1 | 5 | 4 |
| $V_7$ | 5 | 3 | 1 | 2 | 4 | 5 | 1 | 2 | 3 | 3 | 3 | 1 | 2 | 3 | 3 |

Table. 2. Levels of $N_i(C_j)$ values assigned to the $C_j$ criteria on a Likert scale form 1 to 5

| Situations | $p(S_1)=0,4$ | | | | | $p(S_2)=0,5$ | | | | | $p(S_3)=0,1$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Criterions of decision** | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
| *Coefficients of importance* | *0,3* | *0,2* | *0,2* | *0,15* | *0,15* | *0,3* | *0,2* | *0,2* | *0,15* | *0,15* | *0,3* | *0,2* | *0,2* | *0,15* | *0,15* |
| $V_1$ | 0,48 | 0,40 | 0,24 | 0,18 | 0,24 | 0,60 | 0,10 | 0,30 | 0,08 | 0,23 | 0,12 | 0,10 | 0,06 | 0,05 | 0,10 |
| $V_2$ | 0,36 | 0,24 | 0,40 | 0,24 | 0,30 | 0,45 | 0,30 | 0,50 | 0,15 | 0,30 | 0,09 | 0,06 | 0,10 | 0,10 | 0,05 |
| $V_3$ | 0,60 | 0,32 | 0,24 | 0,30 | 0,06 | 0,45 | 0,40 | 0,30 | 0,23 | 0,38 | 0,09 | 0,08 | 0,06 | 0,20 | 0,15 |
| $V_4$ | 0,48 | 0,32 | 0,32 | 0,06 | 0,03 | 0,60 | 0,30 | 0,40 | 0,30 | 0,15 | 0,12 | 0,04 | 0,08 | 0,15 | 0,20 |
| $V_5$ | 0,60 | 0,24 | 0,40 | 0,03 | 0,06 | 0,75 | 0,30 | 0,50 | 0,38 | 0,08 | 0,15 | 0,06 | 0,10 | 0,20 | 0,25 |
| $V_6$ | 0,30 | 0,24 | 0,32 | 0,18 | 0,18 | 0,45 | 0,20 | 0,40 | 0,30 | 0,15 | 0,12 | 0,04 | 0,02 | 0,25 | 0,20 |
| $V_7$ | 0,60 | 0,24 | 0,08 | 0,06 | 0,24 | 0,75 | 0,10 | 0,30 | 0,23 | 0,23 | 0,09 | 0,02 | 0,04 | 0,15 | 0,15 |

Table. 2. The decision matrix for the proposed multicriterial problem

| Situations | $p(S_1)=0,4$ | | | | | $p(S_2)=0,5$ | | | | | $p(S_3)=0,1$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Criterions of decision** | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
| *Coefficients of importance* | *0,3* | *0,2* | *0,2* | *0,15* | *0,15* | *0,3* | *0,2* | *0,2* | *0,15* | *0,15* | *0,3* | *0,2* | *0,2* | *0,15* | *0,15* |
| $V_1$ | 0,50 | 1,00 | 0,00 | 0,56 | 0,78 | 0,50 | 0,00 | 0,00 | 0,00 | 0,50 | 0,50 | 1,00 | 0,00 | 0,00 | 0,25 |
| $V_2$ | 0,00 | 0,00 | 1,00 | 0,78 | 1,00 | 0,00 | 0,67 | 1,00 | 0,25 | 0,75 | 0,00 | 0,33 | 1,00 | 0,33 | 0,00 |
| $V_3$ | 1,00 | 0,50 | 0,00 | 1,00 | 0,11 | 0,00 | 1,00 | 0,00 | 0,50 | 1,00 | 0,00 | 0,67 | 0,00 | 1,00 | 0,50 |
| $V_4$ | 0,50 | 0,50 | 0,50 | 0,11 | 0,00 | 0,50 | 0,67 | 0,50 | 0,75 | 0,25 | 0,50 | 0,00 | 0,50 | 0,67 | 0,75 |
| $V_5$ | 1,00 | 0,00 | 1,00 | 0,00 | 0,11 | 1,00 | 0,67 | 1,00 | 1,00 | 0,00 | 1,00 | 0,33 | 1,00 | 1,00 | 1,00 |
| $V_6$ | 0,00 | 0,00 | 0,50 | 0,56 | 0,56 | 0,00 | 0,00 | 0,50 | 0,67 | 0,25 | 0,50 | 0,00 | 0,00 | 1,00 | 0,75 |
| $V_7$ | 0,00 | 1,00 | 0,00 | 0,00 | 0,00 | 0,50 | 1,00 | 0,00 | 0,00 | 0,00 | 0,00 | 0,75 | 1,00 | 1,00 | 1,00 |

Table. 3. Relevance matrices for the proposed multicriterial problem

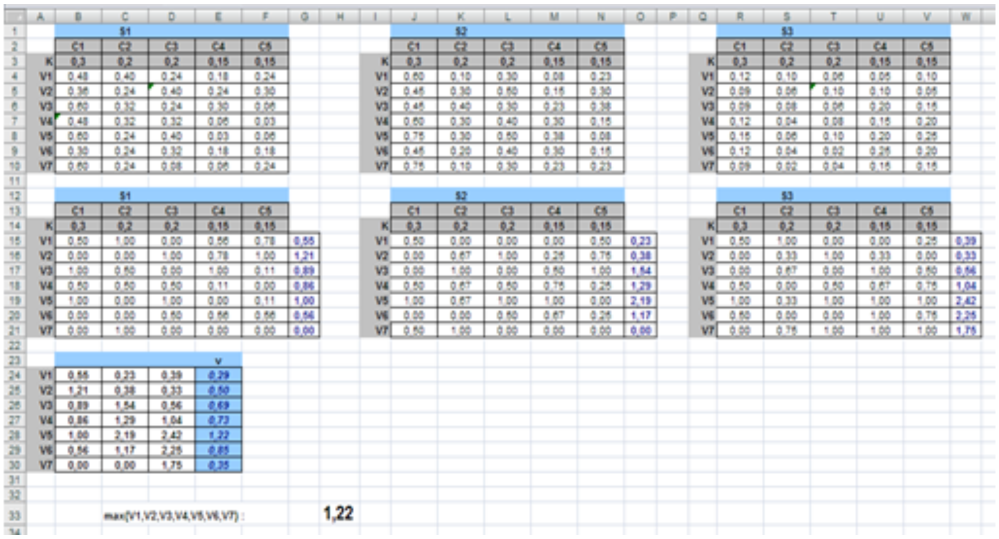
| | | | |
|---|---|---|---|
| $V_1$ | 0,55 | 0,23 | 0,39 |
| $V_2$ | 1,21 | 0,38 | 0,33 |
| $V_3$ | 0,89 | 1,54 | 0,56 |
| $V_4$ | 0,86 | 1,29 | 1,04 |
| $V_5$ | 1,00 | 2,19 | 2,42 |
| $V_6$ | 0,56 | 1,17 | 2,25 |
| $V_7$ | 0,00 | 0,00 | 1,75 |

Table. 4. The synthetic relevance matrix for the proposed multicriterial problem

| | | | | |
|---|---|---|---|---|
| $V_1$ | 0,55 | 0,23 | 0,39 | 0,29 |
| $V_2$ | 1,21 | 0,38 | 0,33 | 0,50 |
| $V_3$ | 0,89 | 1,54 | 0,56 | 0,69 |
| $V_4$ | 0,86 | 1,29 | 1,04 | 0,73 |
| $V_5$ | 1,00 | 2,19 | 2,42 | 1,22 |
| $V_6$ | 0,56 | 1,17 | 2,25 | 0,85 |
| $V_7$ | 0,00 | 0,00 | 1,75 | 0,35 |

Table. 5. Levels assigned to the variables (types of water) for the proposed multicriterial problem

Should the value levels assigned to the $V_i, i = 1, .., 5$ alternatives on a Likert scale form 1 to 5, in relation to the $C_j$ criterion, i.e. $N_i(C_j)$, the $kj$ relevance coefficients and the occurrence probabilities of the $p(S_h)$ situation, then the optimum alternative selected by means of the multicriterial decision would be different. Therefore, its identification depends on estimates and the managerial expertise in interpreting the problem data.



| S1 | C1 | C2 | C3 | C4 | C5 |
|---|---|---|---|---|---|
| K | 0,3 | 0,2 | 0,2 | 0,15 | 0,15 |
| V1 | 0,48 | 0,40 | 0,24 | 0,18 | 0,24 |
| V2 | 0,36 | 0,24 | 0,40 | 0,24 | 0,30 |
| V3 | 0,60 | 0,32 | 0,24 | 0,30 | 0,06 |
| V4 | 0,48 | 0,32 | 0,32 | 0,06 | 0,03 |
| V5 | 0,60 | 0,24 | 0,40 | 0,03 | 0,06 |
| V6 | 0,30 | 0,24 | 0,32 | 0,18 | 0,18 |
| V7 | 0,60 | 0,24 | 0,08 | 0,06 | 0,24 |

| S2 | C1 | C2 | C3 | C4 | C5 |
|---|---|---|---|---|---|
| K | 0,3 | 0,2 | 0,2 | 0,15 | 0,15 |
| V1 | 0,60 | 0,10 | 0,30 | 0,08 | 0,23 |
| V2 | 0,45 | 0,30 | 0,60 | 0,15 | 0,30 |
| V3 | 0,45 | 0,40 | 0,30 | 0,23 | 0,38 |
| V4 | 0,60 | 0,30 | 0,40 | 0,30 | 0,15 |
| V5 | 0,75 | 0,30 | 0,60 | 0,38 | 0,08 |
| V6 | 0,45 | 0,20 | 0,40 | 0,30 | 0,15 |
| V7 | 0,75 | 0,10 | 0,30 | 0,23 | 0,23 |

| S3 | C1 | C2 | C3 | C4 | C5 |
|---|---|---|---|---|---|
| K | 0,3 | 0,2 | 0,2 | 0,15 | 0,15 |
| V1 | 0,12 | 0,10 | 0,06 | 0,05 | 0,10 |
| V2 | 0,09 | 0,06 | 0,10 | 0,10 | 0,05 |
| V3 | 0,09 | 0,08 | 0,06 | 0,20 | 0,15 |
| V4 | 0,12 | 0,04 | 0,08 | 0,15 | 0,20 |
| V5 | 0,15 | 0,06 | 0,10 | 0,20 | 0,25 |
| V6 | 0,12 | 0,04 | 0,02 | 0,25 | 0,20 |
| V7 | 0,09 | 0,02 | 0,04 | 0,15 | 0,15 |

| S1 | C1 | C2 | C3 | C4 | C5 | |
|---|---|---|---|---|---|---|
| K | 0,3 | 0,2 | 0,2 | 0,15 | 0,15 | |
| V1 | 0,50 | 1,00 | 0,00 | 0,56 | 0,78 | 0,55 |
| V2 | 0,00 | 0,00 | 1,00 | 0,78 | 1,00 | 1,21 |
| V3 | 1,00 | 0,50 | 0,00 | 1,00 | 0,11 | 0,89 |
| V4 | 0,50 | 0,50 | 0,50 | 0,11 | 0,00 | 0,86 |
| V5 | 1,00 | 0,00 | 1,00 | 0,00 | 0,11 | 1,00 |
| V6 | 0,00 | 0,00 | 0,50 | 0,56 | 0,56 | 0,56 |
| V7 | 0,00 | 1,00 | 0,00 | 0,00 | 0,00 | 0,00 |

| S2 | C1 | C2 | C3 | C4 | C5 | |
|---|---|---|---|---|---|---|
| K | 0,3 | 0,2 | 0,2 | 0,15 | 0,15 | |
| V1 | 0,50 | 0,00 | 0,00 | 0,00 | 0,50 | 0,23 |
| V2 | 0,00 | 0,67 | 1,00 | 0,25 | 0,75 | 0,38 |
| V3 | 0,00 | 1,00 | 0,00 | 0,50 | 1,00 | 1,54 |
| V4 | 0,50 | 0,67 | 0,50 | 0,75 | 0,25 | 1,29 |
| V5 | 1,00 | 0,67 | 1,00 | 1,00 | 0,00 | 2,19 |
| V6 | 0,00 | 0,00 | 0,50 | 0,67 | 0,25 | 1,17 |
| V7 | 0,50 | 1,00 | 0,00 | 0,00 | 0,00 | 0,00 |

| S3 | C1 | C2 | C3 | C4 | C5 | |
|---|---|---|---|---|---|---|
| K | 0,3 | 0,2 | 0,2 | 0,15 | 0,15 | |
| V1 | 0,50 | 1,00 | 0,00 | 0,00 | 0,25 | 0,39 |
| V2 | 0,00 | 0,33 | 1,00 | 0,33 | 0,00 | 0,33 |
| V3 | 0,00 | 0,67 | 0,00 | 1,00 | 0,50 | 0,56 |
| V4 | 0,50 | 0,00 | 0,50 | 0,67 | 0,75 | 1,04 |
| V5 | 1,00 | 0,33 | 1,00 | 1,00 | 1,00 | 2,42 |
| V6 | 0,50 | 0,00 | 0,00 | 1,00 | 0,75 | 2,25 |
| V7 | 0,00 | 0,75 | 1,00 | 1,00 | 1,00 | 1,75 |

| | | | | v |
|---|---|---|---|---|
| V1 | 0,55 | 0,23 | 0,39 | 0,29 |
| V2 | 1,21 | 0,38 | 0,33 | 0,50 |
| V3 | 0,89 | 1,54 | 0,56 | 0,69 |
| V4 | 0,86 | 1,29 | 1,04 | 0,73 |
| V5 | 1,00 | 2,19 | 2,42 | 1,22 |
| V6 | 0,56 | 1,17 | 2,25 | 0,85 |
| V7 | 0,00 | 0,00 | 1,75 | 0,35 |

max(V1,V2,V3,V4,V5,V6,V7) : 1,22

Fig. 1. Application in Microsoft Excel

## 4. Quality of Drinking Water in Sibiu County([1],[5])

(i) According to the analysis results, for the quality of the drinking water, available on the website of the S.C. Ap Canal S.A. Sibiu-Romania, here is the total amount of calcium and magnesium, pH, turbidity, nitrite concentration, colour, for the water provided by the Chlorination Stations in Sibiu county, throughout the year 2010 (Table 6)

A comparative graphic analysis performed in fig. 2 shows that the drinking water provided by the Avrig Water Treatment Station has the highest calcium and magnesium concentration, therefore it is preferred by those people lacking these minerals, and highly recommended compared to the ones included in table 6 for those suffering from hepatobiliar diseases, thyroid insufficiency, neurosis, hyperacid gastritis, peptic ulcer, rickets, osteomalacia, osteoporosis, muscle cramps, palpitations:

The calcium and magnesium deficit as well as the low value sof drinking water hardness represent risk factors in cases of morbidity entailed by cardiovascular diseases.

Likewise, the comparative graphic analysis in fig.3 shows that the drinking water provided by the Avrig Water Treatment Station has the highest level of pH, and therefore the most alkaline of all types of water included in table 6 and thus preferred by those consumers who wish to maintain a low level of acidity in their body, since a high acidity may also cause various types of cancer:

Chlorinated water provided to the consumer by Cisnădie Water Treatment Station has the highest level of turbidity of all types of drinking water included in table 6, therefore the highest concentration of fine particles that may not be easily noticed, which however may diffuse and reflect light when they are in suspension; thus this type of drinking water has a poorer quality than other types of water in the table.

Chlorinated water from provided ot the consumer by Cisnădie Water Treatment Station has the highest level of nitrites of all types of drinking water included in table 6, therefore it also has the highest toxicity compared to the other ones. Consequently, this reinforces the idea that this type of drinking water is poorer that other types of water in the table.

According to specialized studies, high concentrations of chlorine entail organoleptic changes. Any deviation of organoleptic indicators from health norms has serious implications of consumers‘ psyche, whereas water consumption free of any satisfaction will not quench thirst.

Of all types of drinking water included in table 6, the one provided by Cisnădie Water Treatment Station has the highest level of residual chlorine, which makes it poor in quality, as shown in fig. 6 and 7.

An analysis the correlation among various quality indicators of drinking water will provide details about the quality of drinking water. Thus, according to table 7, the correlation coefficient

| Types of drinking water | Calcium and magnesium concentration ($DH^0$) | pH(units) | Turbidity (NTU) | Nitrites (mg/l) | Residual chlorine (mg/l) | Colour ($m^{-1}$) | Place |
|---|---|---|---|---|---|---|---|
| $V_1$ | 0,88 | 6,58 | 2,16 | 0,0020 | 0,41 | 0,41 | Chlorinated water provided to the consumer by Cisnădie Water Treatment Station |
| $V_2$ | 1,39 | 6,59 | 1,32 | 0,0010 | 0,21 | 0,21 | Chlorinated water provided to the consumer by Lunca Ştezii Water Treatment Station |
| $V_3$ | 0,93 | 6,77 | 1,16 | 0,0011 | 0,31 | 0,21 | Chlorinated water provided to the consumer by Dumbrava Water Treatment Station |
| $V_4$ | 2,41 | 7,33 | 0,45 | 0,0012 | 0,32 | 0,31 | Chlorinated water provided to the consumer by Avrig Water Treatment Station |
| Valori admise | 5 | 6,5-9,5 | 5 | 0,5 | 0,5 | 0,5 | |
| Standard for methods | SR ISO 6059/2008 | SR ISO 10523/2009 | SR EN ISO 7027/2001 | SR ISO 7150-1/2001 | STAS 6364/1978 | SR EN ISO 7887/2002 | |

Table. 6. Types of drinking water in Sibiu county

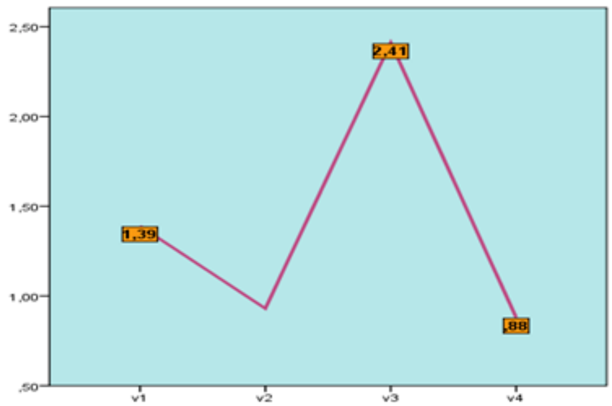



Fig. 2. Calcium and magnesium concentration

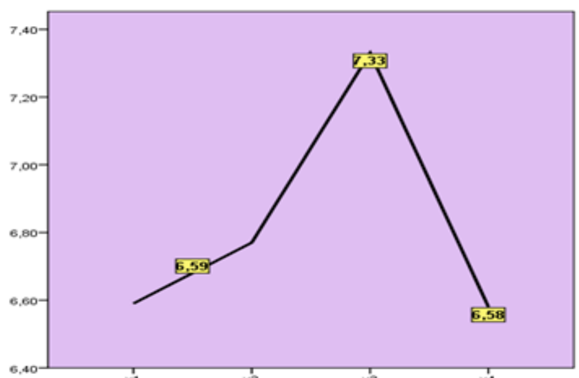



Fig. 3. pH values

between the indicators ”calcium and magnesium concentration” and ”residual chlorine” is -0.235 which indicates a negative correlation, of low intensity, among the two indicators. The increase of one indicators entails a diminishing of the other one.

According to table 8, the correlation coefficient between ”residual chlorine” and ”nitrites” concentration is 0.880 which shows a positive correlation, of high intensity, between the two indicators. The increase of one indicators entails, to a great extent, an increase of the other one.

According to table 9, the correlation coefficient between ”colour” and ”turbidity” indicators is 0.488 which shows a positive correlation, of moderate intensity, between the two indicators. The increase of one indicators entails a moderate impact on the increase of the other one.

(a) We have selected the following criteria for the types of drinking water:

$C_1$ = calcium and magnesium concentration (DH0);
$C_2$ = pH(units);

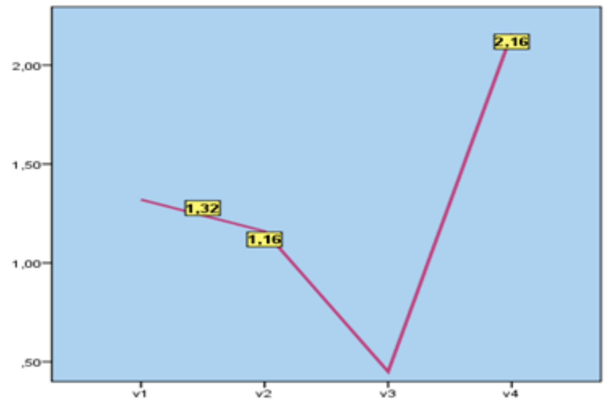



Fig. 4. Turbidity of drinking water samples

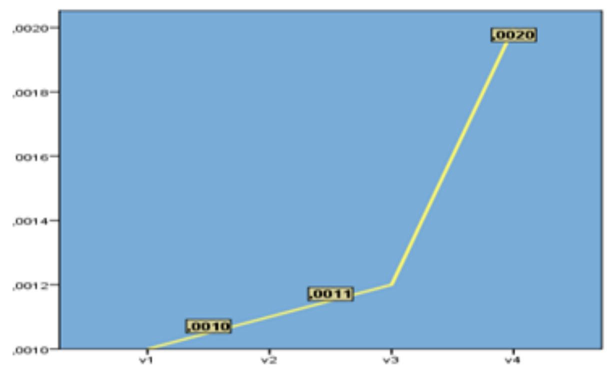



Fig. 5. Nitrites concentrations

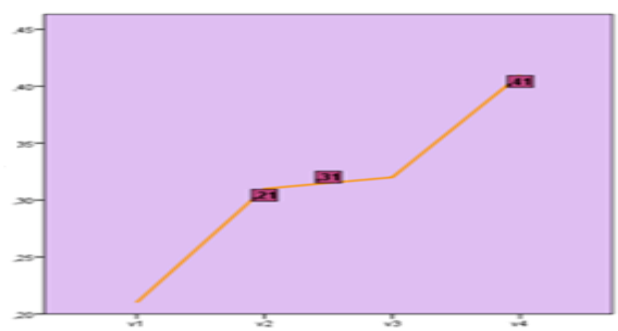


Fig. 6. Residual chlorine concentration

$C_3$ = turbidity (NTU);
$C_4$ = nitrites (mg/l);
$C_5$ = residual chlorine (mg/l);
$C_6$ = colour (m-1).

(b) We assume that the criteria relevance coefficients are: $k_1 = 0,1, k_2 = 0,3, k_3 = 0,2, k_4 = 0,2, k_5 = 0,2, k_6 = 0,1$.

(c) The types of drinking water subject to quality multicriterial analysis are:

$V_1$ = chlorinated water provided to the consumer by Cisnădie Water Treatment Station;

$V_2$ chlorinated water provided to the consumer by Lunca Stezii Water

$V_3$ chlorinated water provided to the consumer by Dumbrava Water Treatment Station;

$V_4$ chlorinated water provided to the consumer by Avrig Water Treatment Station.

The $a_{ij}$ inside the matrices in table 1 represent

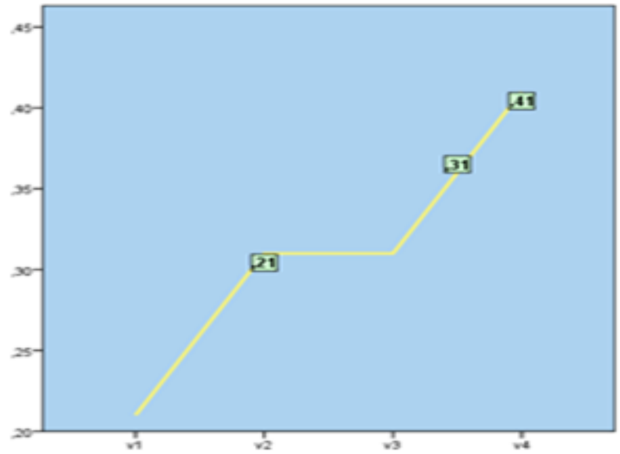



Fig. 7. Drinking water sample colour

| Correlations | | | |
|---|---|---|---|
| | | calcium and magnesium concentration | residual chlorine |
| calcium and magnesium concentration | Pearson Correlation | 1,000 | -,235 |
| | Sig. (2-tailed) | | ,765 |
| | N | 4,000 | 4 |
| residual chlorine | Pearson Correlation | -,235 | 1,000 |
| | Sig. (2-tailed) | ,765 | |
| | N | 4 | 4,000 |

Table. 7. Correlation coefficient between the indicators "calcium and magnesium concentration" and "residual chlorine"

| Correlations | | | |
|---|---|---|---|
| | | residual chlorine | nitrites |
| residual chlorine | Pearson Correlation | 1,000 | ,880 |
| | Sig. (2-tailed) | | ,120 |
| | N | 4,000 | 4 |
| nitrites | Pearson Correlation | ,880 | 1,000 |
| | Sig. (2-tailed) | ,120 | |
| | N | 4 | 4,000 |

Table. 8. Correlation coefficient between "residual chlorine" and "nitrites" indicators

| Correlations | | | |
|---|---|---|---|
| | | colour | turbidity |
| colour | Pearson Correlation | 1,000 | ,488 |
| | Sig. (2-tailed) | | ,512 |
| | N | 4,000 | 4 |
| turbidity | Pearson Correlation | ,488 | 1,000 |
| | Sig. (2-tailed) | ,512 | |
| | N | 4 | 4,000 |

Table 9. Correlation coefficient between colour and turbidity coefficients

here a product of the values assigned to the $C_j$ criterion on a Likert scale from 1 to 5, i.e. $N_i(C_j)$ and the $k_j$ relevance coefficients. This product is calculated by the formula:

$$a_{ij} = N_i(C_j)k_j \tag{5}$$

| Criterions of decision | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ |
|---|---|---|---|---|---|---|
| *Coefficients of importance* | *0,1* | *0,3* | *0,2* | *0,2* | *0,2* | *0,1* |
| **$V_1$** | 5 | 5 | 4 | 2 | 4 | 3 |
| **$V_2$** | 5 | 1 | 3 | 1 | 2 | 5 |
| **$V_3$** | 3 | 5 | 4 | 3 | 3 | 4 |
| **$V_4$** | 1 | 5 | 1 | 5 | 2 | 1 |

Table 10. Levels of Ni(Cj) values assigned to the $C_j$ criteria on a Likert scale from 1 to 5

where $i = 1, 2, 3, 4, 5,\ j = 1, 2, 3$.

This is a subjective assessment of values and it is determined by the expertise of the specialists in charge of drinking water quality:

The $a_{ij}$ elements are automatically calculated and displayed by the Microsoft Excel software (fig.7)

Based on decsion matrices on tables 10 and 11, several decision methods and criteria may be applied, scuh as the "mathematical hope" method from utility theory, which we have also employed in the present paper (see also section 2).

In this respect, let us proceed to change all consequences in utilities, in keeping with formula (2) resulting in the data in table 12 by means of the Microsoft Excel software:

We have calculated the synthetic utility for each type of drinkig water, according to the formula (3), and further to the synthetic utilities we have designed a new matrix by means of Microsoft Excel:

To conclude, the final column shows that the qualitative hierarchy of drinking water is:

(a) $V_4$ chlorinated water provided to the consumer by Avrig Water Treatment Station = optimum version;

(b) $V_2$ chlorinated water provided to the consumer by Lunca Ştezii Water Treatment Station;

(c) $V_3$ chlorinated water provided to the consumer by Dumbrava Water Treatment Station;

(d) $V_1$ chlorinated water provided to the consumer by Cisnădie Water Treatment Station;

We have applied the utility theory, similarly to section 2, and performed the model simulation by means of Microsoft Excel, in order to obtain the results given in figure 8:

The concluding remark shows that the $V_4$ alternative is the optimum one, hence the $V_4$ type of drinking water has the highest quality, given the circumstances that the selection of $a_{ij}$ initial elements has been influenced by the assessment and expertise of managers and organizers.

$$1)\ V4 \quad 2)\ V2 \quad 3)\ V3 \quad 4)\ V1 \qquad (6)$$

Should the value levels assigned to the $V_i$ $i = 1, .., 5$ alternatives on a Likert scale form 1 to 5, in relation to the $C_j$ criterion, i.e. $N_i(C_j)$, the $k_j$ relevance coefficients and the occurrence probabilities of the $p(S_h)$ situation, then the optimum alternative selected by means of the multicriterial decision would be different. Therefore, its identification depends on estimates and the managerial expertise in interpreting the problem data.

## Conclusions

All specialists claim that mathematical reasoning creates possibilities of understanding and studying of the problems regarding nature, life and society. In the paper's case, we showed once more that this can also be used for the problem of analyzing the quality of drinkable water, which represents a problem of very high importance in every country of the world, in the context of increasing the world's population and of sustainable development. Moreover, it is noticed in the paper that software such as Microsoft Excel and SPSS can be used for simulating the mathematical models that can be applied for the analysis of the quality of drinkable water and for the management of water quality.

| Criterions of decision | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ |
|---|---|---|---|---|---|---|
| *Coefficients of importance* | *0,1* | *0,3* | *0,2* | *0,2* | *0,2* | *0,1* |
| $V_1$ | 0,5 | 1,5 | 0,8 | 0,4 | 0,8 | 0,3 |
| $V_2$ | 0,5 | 0,3 | 0,6 | 0,2 | 0,4 | 0,5 |
| $V_3$ | 0,3 | 1,5 | 0,8 | 0,6 | 0,6 | 0,4 |
| $V_4$ | 0,1 | 1,5 | 0,2 | 1,0 | 0,4 | 0,1 |

Table 11. Decision matrix for the proposed multicriterial problem

| Criterions of decision | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ | $C_6$ |
|---|---|---|---|---|---|---|
| *Coefficients of importance* | *0,1* | *0,3* | *0,2* | *0,2* | *0,2* | *0,1* |
| $V_1$ | 1,0 | 1,0 | 1,0 | 0,25 | 0,67 | 0,0 |
| $V_2$ | 1,0 | 0,0 | 0,67 | 0,0 | 0,0 | 0,29 |
| $V_3$ | 0,5 | 1,0 | 1,0 | 0,5 | 0,33 | 0,14 |
| $V_4$ | 0,5 | 1,0 | 0,0 | 1,0 | 1,0 | 1,0 |

Table 12. Utility matrices for the proposed multicriterial problem

| | |
|---|---|
| $V_1$ | 0,78 |
| $V_2$ | 1,67 |
| $V_3$ | 1,21 |
| $V_4$ | 1,98 |

Table 13. Levels associated to the types of water in the proposed multicriterial problem

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| | C1 | C2 | C3 | C4 | C5 | C6 |
|---|---|---|---|---|---|---|
| **K** | **0,1** | **0,3** | **0,2** | **0,2** | **0,2** | **0,1** |
| **V1** | 0,50 | 1,50 | 0,80 | 0,40 | 0,80 | 0,30 |
| **V2** | 0,50 | 0,30 | 0,60 | 0,20 | 0,40 | 0,50 |
| **V3** | 0,30 | 1,50 | 0,80 | 0,60 | 0,60 | 0,40 |
| **V4** | 0,10 | 1,50 | 0,20 | 1,00 | 1,00 | 1,00 |

| | C1 | C2 | C3 | C4 | C5 | C6 | |
|---|---|---|---|---|---|---|---|
| **K** | **0,1** | **0,3** | **0,2** | **0,2** | **0,2** | **0,1** | |
| **V1** | 1,00 | 1,00 | 1,00 | 0,25 | 0,67 | 0,00 | **0,78** |
| **V2** | 1,00 | 0,00 | 0,67 | 0,00 | 0,00 | 0,29 | **1,67** |
| **V3** | 0,50 | 1,00 | 1,00 | 0,50 | 0,33 | 0,14 | **1,21** |
| **V4** | 0,00 | 1,00 | 0,00 | 1,00 | 1,00 | 1,00 | **1,98** |

| | |
|---|---|
| **V1** | **0,78** |
| **V2** | **1,67** |
| **V3** | **1,21** |
| **V4** | **1,98** |

**max{V1,V2,V3,V4} este** **1,98**

Fig. 8. Simulation in Microsoft Excel