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Null Principal Directions and Canonical Vectors

Direcciones principales nulas y vectores canónicos

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Abstract

We exhibit expressions to obtain the Debever-Penrose vectors from a given null canonical tetrad.

***Keywords*:** Debever-Penrose directions, Petrov classification, Weyl tensor, Newman-Penrose formalism.

Resumen

Se muestran expresiones para obtener los vectores de Debever-Penrose a partir de una tétrada canónica dada.

***Palabras Claves*:** Direcciones de Debever-Penrose, Clasificación de Petrov, Tensor de Weyl, Formalismo de Newman-Penrose.

1. Introduction

At any event of R_4 there is a Newman-Penrose (NP) [1,2] null canonical tetrad (NCT) [3–5] $(m^r, \bar{m}^r, l^r, n^r)$ such that the NP components of Weyl tensor take the following values for each Petrov type [6] of spacetime:

$$\begin{aligned}
&\mathrm{N}: \psi_4 = 1,\ \psi_a = 0,\ a \neq 4\\
&\mathrm{III}: \psi_3 = -i/\sqrt{2},\ \psi_b = 0,\ b \neq 3\\
&\mathrm{D}: \psi_2 = \lambda_1,\ \psi_c = 0,\ c \neq 2\\
&\mathrm{II}: \psi_2 = \lambda_1,\ \psi_4 = 1,\ \ \psi_j = 0,\ j \neq 2, 4\\
&\mathrm{I}: \psi_o = \psi_4 = (\lambda_2 - \lambda_1)/2, \psi_2 = -\lambda_3/2,\\
&\psi_r = 0, r = 1, 3
\end{aligned} \tag{1}$$

where $\lambda_a, a = 1, 2, 3$ are the eigenvalues of the corresponding 3×3 complex matrix of Petrov [7–11]

Robinson [12] comments that Cartan [13] was the first to mention the existence of the null real vectors, now named Debever-Penrose (DP) principal directions [14–19], which are very important [5] in general relativity. Then, it is interesting to learn how to construct the DP vectors at an event where a NCT verifying (1) is given. For the Petrov types N, D and III is known [4, 5, 17, 19] that the DP directions K_a coincide with l^r or/and n^r of the NCT, thus, here only the explicit relationship of K_a with the NCT for the cases I and II will be exhibited.

2. Petrov type II

There are three DP null real vectors $\underset{c}{K_r}$ that may be obtained from NTC via:

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$$\underset{j}{K_r} = n_r + \gamma_j \bar{\gamma}_j l_r + i(\gamma_j \bar{m}_r - \bar{\gamma}_j m_r), \quad j = 1, 2, \ \underset{3}{K_r} = n_r, \tag{2}$$

where γ_j are the two roots of $\gamma^2 = 6\lambda_1$. This relation (2) represents a special case of the general equation for the null principal directions — [20]:

$$K_r = n_r + b\bar{b}l_r + b\bar{m}_r + \bar{b}m_r, \tag{3}$$

setting $b = i\gamma$ and substituting the corresponding values of the Weyl scalars ψ_c into the equation (3.72) of [20]:

$$\psi_4 b^4 + 4\psi_3 b^3 + 6\psi_2 b^2 + 4\psi_1 b + \psi_0 = 0, \tag{4}$$

one gets the values of b and γ, then comes straightforwardly to (2)

$$\underset{c}{K^r} \underset{c}{K} {}_{[p} W_{q]} ra \, {}_{[j} \underset{c}{K}_{t]} \underset{c}{K^a} = 0, \tag{5}$$

that any DP vector must satisfy with $W_{abjr} = C_{abjr} + {}^*C_{abjr}$.

3. Petrov Type I

Here we have four DP principal directions $\underset{11}{K_r}$, $\underset{12}{K_r}$, $\underset{21}{K_r}$ and $\underset{22}{K_r}$ whose expressions in function of the NCT are given by:

$$\underset{jc}{K_r} = n_r - \frac{\beta_c(1 + i\bar{\varphi}_j)}{\bar{\beta}_c(1 + i\varphi_j)} l_r + \frac{1 - i\varphi_j}{\beta_c} \bar{m}_r + \frac{1 + i\bar{\varphi}_j}{\bar{\beta}_c} m_r, \tag{6}$$

$j, c = 1, 2$ where β_c and φ_j are the roots of:

$$\beta^2 = (\lambda_2 - \lambda_1)/(\lambda_3 - \lambda_2), \quad \varphi^2 = (\lambda_1 - \lambda_3)/(\lambda_3 - \lambda_2), \tag{7}$$

and we must remember that for Petrov type I the eigenvalues λ_j are different each other with the restriction $\lambda_1 + \lambda_2 + \lambda_3 = 0$; as in the previous case, the null vectors (6) satisfy (5).

4. Conclusions

It is no difficult to show that the vectors (6) have the structure (3), however, in the literature we have not explicitly found the expressions (6) which may have interesting applications in general relativity, for example, in the quest of an general formula for the Lanczos potential [21–23] of spacetimes with Petrov types I and II.

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