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Gödel spacetime and Lanczos potential

Espacio-tiempo de Gödel y potencial de Lanczos

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Abstract. We exhibit that, in Gödel geometry, the Lanczos generator can be written algebraically in terms of the Weyl tensor.

Keywords. Lanczos potential, Conformal tensor, Gödel's universe.

Resumen. Exhibimos que, en la geometría de Gödel, el generador de Lanczos puede ser escrito algebraicamente en términos del tensor de Weyl.

Palabras Claves. Potencial de Lanczos; tensor conformal; universo de Gödel.

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1. Introduction

Gödel proposed this geometry to represent a rotational universe, under the hypothesis that the cosmos is composed of an incoherent perfect fluid.

In general relativity, the Lanczos spintensor K_{abc} has the symmetries [1, 2]:

$$K_{abc} = -K_{bac}, \qquad K_{abc} + K_{bca} + K_{cab} = 0, \qquad K_a{}^b{}_b = 0, \tag{1}$$

and for the Gödel metric [3–9]:

$$ds^2 = (dx^0)^2 + 2e^{x^3} dx^0 dx^1 + \frac{1}{2} e^{2x^3} (dx^1)^2 - (dx^2)^2 - (dx^3)^2, \tag{2}$$

we have the relation [10–16]:

$$S_{abc} = K_{abc} + \iota\, {}^*K_{abc} = \frac{\iota}{9} [U_{ab} m_c + V_{ab} \bar{m}_c - M_{ab} (l_c + n_c)], \tag{3}$$

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with the canonical null tetrad [17]:

$$(l^\mu) = \frac{1}{\sqrt{2}}(1,0,-1,0), (n^\mu) = \frac{1}{\sqrt{2}}(1,0,1,0), (m^\mu) = (1,-e^{-x^3},0,\frac{\iota}{\sqrt{2}}), (\bar{m}^\mu) = (1,-e^{-x^3},0,-\frac{\iota}{\sqrt{2}}), \quad (4)$$

and :

$$V_{ab} = l_a \ x \ m_b, \qquad U_{ab} = \bar{m}_a \ x_b, \qquad M_{ab} = m_a \ x \ \bar{m}_b + n_a \ x \ l_b. \quad (5)$$

The corresponding expression for the conformal tensor is given by [6], [18]:

$$S_{abcr} \equiv C_{abcr} + \iota^* C_{abcr} = -\frac{1}{3}(M_{ab}M_{cr} + V_{ab}U_{cr} + U_{ab}V_{cr}). \quad (6)$$

In Sec. 2 we exhibit that the Lanczos generator can be written in the form:

$$S_{abc} = S_{abcr}\xi^r, \quad (7)$$

where ξ^r is a complex vector in terms of the canonical null tetrad.

2. Lanczos potential

We know the identity [19–22]:

$$S_{abcr}S^{abct} = \frac{1}{2}(C_2 + \iota^* C_2)\delta^t_r, \quad C_2 = C_{\mu v \alpha\beta}\ C^{\mu v \alpha\beta}, \quad {}^*C_2 = {}^* C_{\mu v \alpha\beta}\ C^{\mu v \alpha\beta}, \quad (8)$$

and for the Gödel geometry (2) we have the values $C_2 = \frac{4}{3}$ and ${}^*C_2 = 0$, then we multiply (7) by S^{abct} and we apply (8) to obtain:

$$\xi^t = \frac{3}{2}S_{abc}S^{abct} = \frac{\iota}{3}(n^t - l^t) = \frac{\sqrt{2}}{3}\iota k^t, \quad (k^\mu) = (0,0,1,0), \quad (k_\mu) = (0,0,-1,0). \quad (9)$$

We note that k^μ is a space-like Killing vector in this Gödel's cosmological model. If we use (9) into (7) we deduce the interesting relations:

$$K_{abc} = -\frac{\sqrt{2}}{3}\ {}^*C_{abcr}k^r, \quad {}^*K_{abc} = \frac{\sqrt{2}}{3}C_{abcr}k^r, \quad (10)$$

that is, the projection of k^μ on the dual of Weyl tensor gives a Lanczos spintensor in Gödel spacetime. The possible physical meaning of the Lanczos potential in general relativity, is an open problem.

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