Journal de Ciencia e Ingeniería, vol.17, no 1, pp.36-48, 2025
http://jci.uniautonoma.edu.co
DOI: https://doi.org/10.46571/JCI.2025.1.4

Corporación Universitaria Autónoma del Cauca
ISSN 2145-2628, e-ISSN 2539-066X
Mathematics

On the Tammes Problem for 60 Points

Sobre el problema de Tammes para 60 puntos

Enrique Antoniano[+1]
[+] **Department of Basic Sciences, Universidad Anahuac, México**

Abstract. In an attempt to solve the Tammes problem for 60 points, we analyzed the positioning obtained by Laszlo Hars [1]. Although we did not achieve the goal, we found a couple of configurations in the induced triangulation that might shed some light on the way to the solution. As a result of this analysis, we achieved a slightly better position where the length of the minimum edge is increased by around $6 \cdot 10^{-9}$ units.

Keywords. Topology; Tammes Problem; Spheres.

Resumen. En un intento por resolver el problema de Tammes para 60 puntos, se analiza la disposición obtenida por Laszlo Hars [1]. Aunque no se alcanza el objetivo, se encontran un par de configuraciones en la triangulación inducida que podrían arrojar un indicio sobre el camino hacia la solución. Como resultado de este análisis, se logra una disposición ligeramente mejor, donde la longitud de la arista mínima se incrementa en aproximadamente $6 \cdot 10^{-9}$ unidades.

Palabras Clave. Topología; Problema de Tammes ; Esferas.

How to Cite. E. Antoniano, On the Tammes Problem for 60 Points, *Jou. Cie. Ing.*, vol. 17, no. 1, pp. 36-48, 2025. doi:10.46571/JCI.2025.1.4

Received: 20/11/2024 **Revised:** 03/02/2025 **Accepted:** 31/03/2025

1. Introduction

The Tammes problem received its name from the Dutch botanist R.M.L. Tammes [2]. He noticed that the pollen grains, almost spherical, have pores distributed in such a way that they appear to be as far apart as possible from each other. Later the problem was mathematized and today we can state it like this: Given a positive integer, how can I place n points on the unit sphere so that they are as far apart as possible from each other? And what is that distance?

To date, solutions are known for n equal to 1 to 14 and for $n = 24$ [3]. In [4], we gave an elementary solution for $n = 4, 6$, and 12 which correspond to the platonic solids based on minimal faces and noticed that for $n = 8$ and 24 which correspond to Archimedean solids, the solution [5] is based on minimal stars which are the set of faces around a vertex. The cases $n = 5$ or 11 are special since they have the same minimal distance as $n = 6$ or 12, respectively. In the other cases [3, 6], we cannot see any pattern to follow.

Then we selected the snub dodecahedron which is an Archimedean with eighty equilateral triangular and twelve pentagonal faces as our candidate to be the solution for sixty points [7]. But we quickly realized that there were better positions due to the work of Laszlo Hars [1], so we proceeded analyzing his solution.

[1] *e-mail: enrique.antoniano@anahuac.mx*

Of course, it is interesting to ask if the Icosidodecahedron, which is an Archimedean with thirty vertices, twenty triangular faces and twelve pentagonal ones [5, 7], is the solution for thirty points.

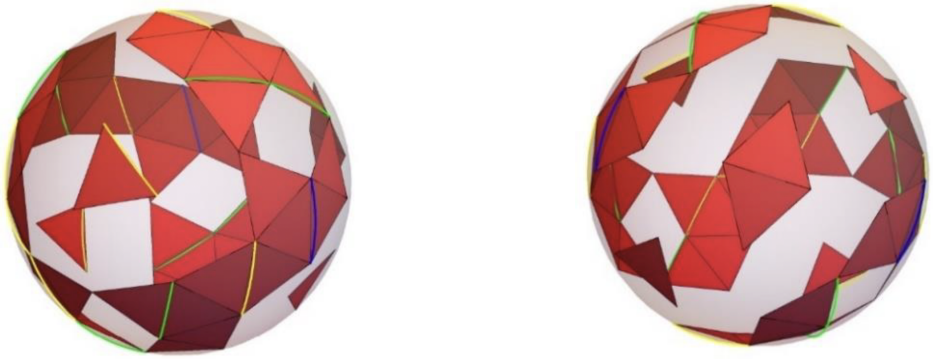
The references are our selection of works related to the Tammes problem, from which we drew inspiration for our research. We strongly recommend reading them.

2. Development

Let us start by defining the distance between two points P and Q on the unit sphere as the angle between the vectors, i.e., $\arccos(P \cdot Q)$.

We took the coordinates of the sixty points generated by Laszlo Hars [1] and input them into Wolfram Mathematica to visualize the induced triangulation. This triangulation consists of vertices (the given points), edges (great circle arcs connecting neighboring points), and faces (spherical triangles).

After multiple attempts to adjust the positions to maximize the shortest edge length, we classified the edges into short (138) and long (36). We observed that the set of vertices of the short edges included all sixty points. By drawing the triangles with these short edges, we identified the following configuration (see Section 3):

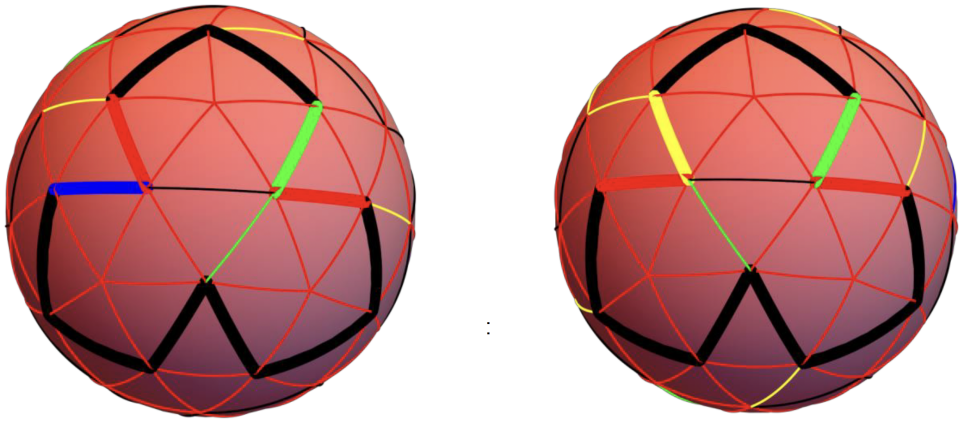


There are two types of polygons: two with six faces and three with ten faces, connected by their vertices, along with four disconnected triangles and a rhombus. To reproduce this structure with fewer edge-length categories, we divided the short edges into 10 categories $A_\alpha, A_{\beta_1}, A_{\beta_2}, A_{\beta_3}, A_{\beta_4}, A_{\gamma_1}, A_{\gamma_2}, A_{\gamma_3}, A_{\gamma_4}, A_\delta$ and one more for the long edges A_L, sorted from smallest to largest assigning average dimensions based on specific ranges. The dimensions obtained for the short are as follows (colors correspond to those used in the illustrations):

$$
\begin{aligned}
\alpha_0 &= \cos^{-1}(0.889473568\ldots) && \text{(The minimal value in the original positioning.)}\\
\alpha &= \alpha_0 + \varepsilon_{c\alpha} && \#(A_\alpha) = 119 \quad \text{(red)}\\
\beta_1 &= \cos^{-1}(0.889318585\ldots) && \#(A_{\beta_1}) = 2 \quad \text{(yellow)}\\
\beta_2 &= \cos^{-1}(0.889273294\ldots) && \#(A_{\beta_2}) = 3 \quad \text{(yellow)}\\
\beta_3 &= \cos^{-1}(0.889100367\ldots) && \#(A_{\beta_3}) = 2 \quad \text{(yellow)}\\
\beta_4 &= \cos^{-1}(0.888781471\ldots) && \#(A_{\beta_4}) = 2 \quad \text{(yellow)}\\
\gamma_1 &= \cos^{-1}(0.888040035\ldots) && \#(A_{\gamma_1}) = 2 \quad \text{(greens)}\\
\gamma_2 &= \cos^{-1}(0.887069585\ldots) && \#(A_{\gamma_2}) = 2 \quad \text{(greens)}\\
\gamma_3 &= \cos^{-1}(0.886480716\ldots) && \#(A_{\gamma_3}) = 2 \quad \text{(greens)}\\
\gamma_4 &= \cos^{-1}(0.886267050\ldots) && \#(A_{\gamma_4}) = 2 \quad \text{(greens)}\\
\delta &= \cos^{-1}(0.882168068\ldots) && \#(A_\delta) \ = 2 \quad \text{(blues)}
\end{aligned}
$$

Now we emulate the structure of the disconnected polygon with short edges, forcing the lengths to be the closest to the previous ones. Then, we decrease the value of $\cos(\alpha)$ with decrements of $\varepsilon_{c\alpha}$, ranging from 0 to $2.3689 \cdot 10^{-9}$, thus obtaining a better result, though not optimal.

Additionally, in the induced triangulation, we observe two pairs of dodecagons like those shown below in bold lines. Note the size of the edges, indicated by their colors, with black being long edges (see Section 4).



Furthermore, the coordinates of the centroids of the four dodecagons are close to the coordinates of a regular tetrahedron.

Here we want to emphasize that this discovery leads us to think that perhaps by improving the symmetries of this configuration and maybe achieving just one instead of two, we could get closer to the optimum. Also, by making its centroids to form a regular tetrahedron.

3. Conclusions

In this triangulation, only eighteen of the 116 triangles are equilateral, although the triangles with short edges are forty-eight and contain the sixty vertices, which leads us to think that there are too many types of short edges.

If we could reduce the type of short edges, we could have more equilateral triangles, which is good to get closer to the optimal solution.

We should also look for better symmetry in the fundamental dodecagons and place them with their barycenter, or other points, which I cannot think of, at the vertices of the regular dodecahedron.

4. Coordinates of the Obtained Points

Point	x	y	z	Point	x	y	z
Q1	0.000000000	0.000000000	1.000000000	Q31	-0.205490122	-0.777294379	0.594632035
Q2	0.000000000	0.457376221	0.889273295	Q32	-0.029692756	-0.456020944	0.889473574
Q3	0.403073451	0.215333620	0.889473566	Q33	-0.384089030	-0.905824819	0.178765247
Q4	0.379729091	0.632920971	0.674697607	Q34	-0.629884605	-0.623451524	0.463199290
Q5	-0.052905324	0.810477541	0.583375678	Q35	-0.744577951	-0.667504380	-0.006448073
Q6	0.308194088	0.903745376	0.297087024	Q36	-0.914995926	-0.314333116	0.252937044
Q7	-0.425377860	0.555336561	0.714601273	Q37	-0.931994731	-0.290838430	-0.216330372
Q8	-0.455595018	0.826171645	0.331471859	Q38	-0.420891408	-0.860266662	-0.287735457
Q9	-0.097348949	0.992320439	0.076310733	Q39	-0.663520111	-0.534080049	-0.523927059
Q10	0.298506682	0.938842557	-0.171663666	Q40	-0.242046813	-0.686038903	-0.686122412
Q11	-0.494876747	0.858263098	-0.135946532	Q41	-0.433056194	-0.290302804	-0.853338511
Q12	-0.110415217	0.909059778	-0.401769586	Q42	-0.784118643	-0.088711865	-0.614237868
Q13	0.283446879	0.747218602	-0.601100846	Q43	0.022960255	-0.379697759	-0.924825626
Q14	0.658371832	0.678503954	-0.325851062	Q44	-0.143596941	0.055399811	-0.988084399

Point	x	y	z	Point	x	y	z
Q15	0.583459674	0.400847543	-0.706325744	Q45	0.757915112	0.408057206	0.508973476
Q16	0.155037868	0.407662702	-0.899874647	Q46	0.701594916	0.698304016	0.141901640
Q17	0.887730127	0.270722759	-0.372349848	Q47	0.948942705	0.301491190	0.092794422
Q18	0.411851594	0.013997118	-0.911143427	Q48	0.208146176	-0.780764408	-0.589136748
Q19	0.771127323	-0.106426034	-0.627722989	Q49	0.025935806	-0.978985937	-0.202271768
Q20	0.966234123	-0.160298845	-0.201732245	Q50	0.478602958	-0.852239426	-0.211251435
Q21	0.464748743	-0.432221526	-0.772782737	Q51	-0.478675490	-0.342419525	0.808466848
Q22	0.748602194	-0.521937345	-0.408871817	Q52	-0.441169291	0.119190738	0.889473566
Q23	0.841833734	-0.537255159	0.051699689	Q53	-0.786749877	-0.044122224	0.615692992
Q24	0.954037428	-0.134942357	0.267587642	Q54	-0.529580423	0.255325260	-0.808921249
Q25	0.724509755	-0.470591925	0.503615781	Q55	-0.237160748	0.618589710	-0.749067120
Q26	0.739038589	-0.032200571	0.672893072	Q56	-0.633683810	0.592126654	-0.497826129
Q27	0.422354693	-0.335834557	0.841921412	Q57	-0.982922130	0.133446231	0.126713022
Q28	0.521221781	-0.814297513	0.255435735	Q58	-0.790911153	0.400438645	0.462718532
Q29	0.260031172	-0.726266973	0.636333303	Q59	-0.825324302	0.564176661	0.023334344
Q30	0.074747874	-0.961342113	0.265017162	Q60	-0.911560327	0.256376187	-0.321448317

5. Wolfram's program with the original positioning

On it the Excel data on `60 numérico.xlsx` contains the coordinate data given in [4]. In Appendix 1 contains the full code.

Referencias

[1] L. Hars, "Numerical solutions of the tammes problem for up to 60 points." Draft 07: November 6, 202.

[2] E. Mooers, "Tammes's problem," 1994. Disponible en: `http://www.uvm.edu/pdodds/files/papers/others/1994/mooers1994a.pdf`, [Accedido: 22-junio-2017].

[3] O. R. Musin and A. S. Tarsov, "The tammes problem for n = 14," *Experimental Mathematics*, vol. 24, no. 4, pp. 460–468, 2015.

[4] E. Antoniano, "The tammes problem," *Journal de Ciencia e Ingeniería*, vol. 11, no. 1, pp. 72–80, 2019.

[5] "Wikipedia, sólidos arquimedianos." `https://es.wikipedia.org/wiki/S%C3%B3lido_arquimediano`.

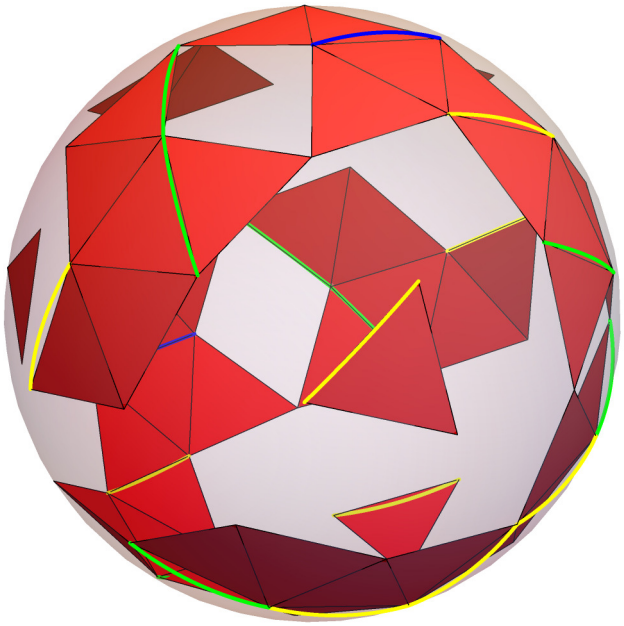
[6] O. R. Musin and A. S. Tarsov, "The strong thirteen spheres problem," *Discrete & Computational Geometry*, vol. 48, no. 1, pp. 128–141, 2012.

[7] "Wikipedia, rombicosahedro." `https://es.wikipedia.org/wiki/Rombicosaedro`.

Appendix 1 - Wolfram's program Code

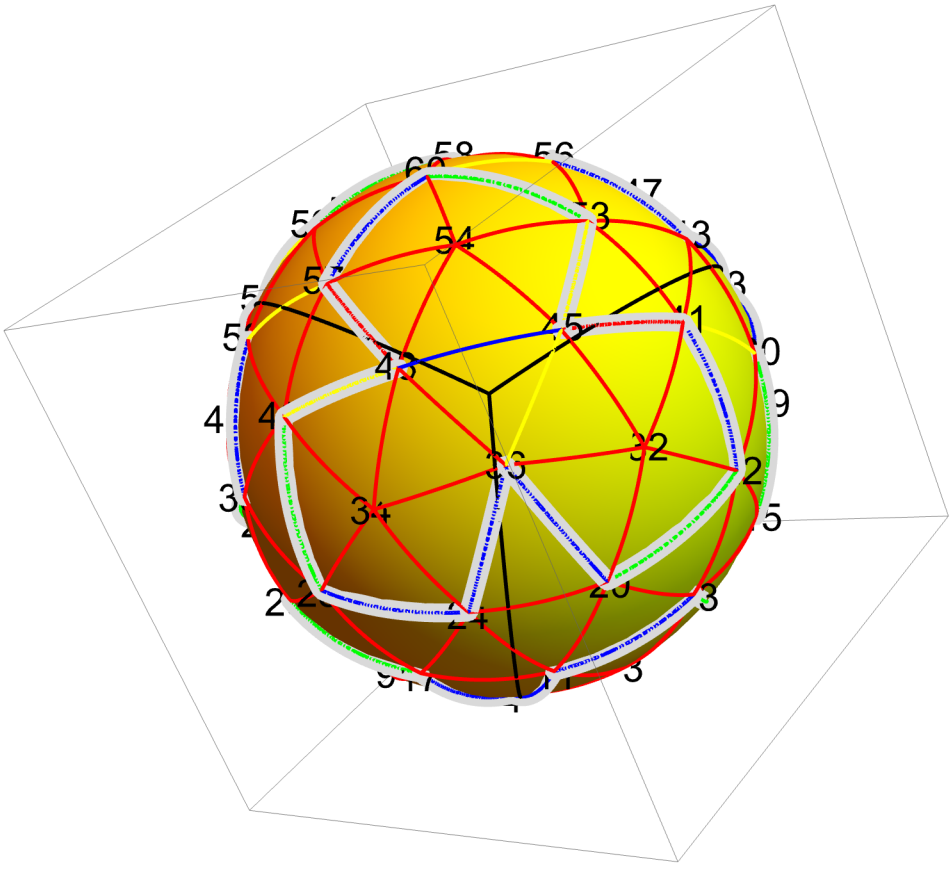
```
Puntos=Flatten[Import["D:\\A\\Anahuac\\Investigaci n y anexas\\Puntos en la esfera\\60
    num rico\\60 num rico.xlsx"],1];
For[i=1,i<=60,i++,P[i]=SetPrecision[Part[Puntos,i],9]]
Pu=Table[P[i],{i,1,60}];

AristasCortas={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[.889473<P[i].P[j],AristasCortas=Union[AristasCortas
    ,{{P[i],P[j]}}]]]]
AristasSemicortas={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[.882168<P[i].P[j]<.889319,AristasSemicortas=Union[
    AristasSemicortas,{{P[i],P[j]}}]]]]
AristasMedianas={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[.840668<P[i].P[j]<.853922,AristasMedianas=Union[
    AristasMedianas,{{P[i],P[j]}}]]]]
AristasLargas={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[.799082<P[i].P[j]<.808467,AristasLargas=Union[
    AristasLargas,{{P[i],P[j]}}]]]]
{Length[AristasCortas],Length[AristasSemicortas],Length[AristasMedianas],Length[
    AristasLargas],Length[AristasCortas]+Length[AristasSemicortas]+Length[AristasMedianas]+
    Length[AristasLargas]}
Out[550]= {119,19,24,12,174}

X=Union[AristasCortas,AristasSemicortas];
Y={};
For[i=1,i<=60,i++,
For[j=i+1,j<=60,j++,For[k=j+1,k<=60,k++,If[MemberQ[X,{P[i],P[j]}]&&MemberQ[X,{P[i],P[k]}]&&
    MemberQ[X,{P[j],P[k]}],Y=Union[Y,{{P[i],P[j],P[k]}}]]]]]
Print["Length[Y]= ",Length[Y]]

AristasSemicortas1={};
AristasSemicortas2={};
AristasSemicortas3={};

For[i=1,i<=19,i++,If[0.8886<Part[AristasSemicortas,i,1].Part[AristasSemicortas,i,2],
    AristasSemicortas1=Union[AristasSemicortas1,{Part[AristasSemicortas,i]}],If[0.886<Part[
    AristasSemicortas,i,1].Part[AristasSemicortas,i,2],AristasSemicortas2=Union[
    AristasSemicortas2,{Part[AristasSemicortas,i]}],AristasSemicortas3=Union[
    AristasSemicortas3,{Part[AristasSemicortas,i]}]]]]
Print["# AristasCortas= ",Length[AristasCortas],", Longitud m nima ",Min[Table[ArcCos[Part[
    AristasCortas,i,1].Part[AristasCortas,i,2]],{i,1,119}]],", Longitud promedio ",Mean[
    Table[ArcCos[Part[AristasCortas,i,1].Part[AristasCortas,i,2]],{i,1,119}]]]
Print["#AristasSemicortas1= ",Length[AristasSemicortas1],", Longitud promedio ",Mean[Table[
    ArcCos[Part[AristasSemicortas1,i,1].Part[AristasSemicortas1,i,2]],{i,1,9}]]]
Print["#AristasSemicortas2= ",Length[AristasSemicortas2],", Longitud promedio ",Mean[Table[
    ArcCos[Part[AristasSemicortas2,i,1].Part[AristasSemicortas2,i,2]],{i,1,8}]]]
Print["#AristasSemicortas3= ",Length[AristasSemicortas3],", Longitud promedio ",Mean[Table[
    ArcCos[Part[AristasSemicortas3,i,1].Part[AristasSemicortas3,i,2]],{i,1,2}]]]


Show[Graphics3D[{Red,Table[Triangle[Part[Y,i]],{i,1,Length[Y]}],White,Opacity[.3],Sphere[](*
    , Opacity[1],Black,Table[Inset[Text[Style[i,20]],(1.1)P[i]],{i,1,60}]*)},Boxed->False],
    ParametricPlot3D[Table[Arc[Part[AristasSemicortas1,i],t],{i,1,9}],{t,0,1},PlotStyle->
    Yellow],ParametricPlot3D[Table[Arc[Part[AristasSemicortas2,i],t],{i,1,8}],{t,0,1},
    PlotStyle->Green],ParametricPlot3D[Table[Arc[Part[AristasSemicortas3,i],t],{i,1,2}],{t
    ,0,1},PlotStyle->Blue]]

```

```
35
36 Durante la evaluaci\363n de In[551]:= Length[Y]= 48
37 Durante la evaluaci\363n de In[551]:= # AristasCortas= 119, Longitud m nima 0.47460441,
       Longitud promedio 0.47460442
38 Durante la evaluaci\363n de In[551]:= #AristasSemicortas1= 9, Longitud promedio 0.47547195
39 Durante la evaluaci\363n de In[551]:= #AristasSemicortas2= 8, Longitud promedio 0.48006082
40 #AristasSemicortas3= 2, Longitud promedio 0.49035004
41
42 Out[563]=
```



```
1  Arc[{A_,B_},t_]:=((1-t)A+t B)/Norm[((1-t)A+t B)]
2  Poligono1={{P[1],P[2]},{P[2],P[12]},{P[12],P[10]},{P[10],P[7]},{P[7],P[16]},{P[16],P[25]},{P
       [25],P[17]},{P[17],P[4]},{P[4],P[11]},{P[11],P[13]},{P[13],P[6]},{P[6],P[1]}};
3  Poligono2={{P[36],P[24]},{P[24],P[28]},{P[28],P[46]},{P[46],P[48]},{P[48],P[57]},{P[57],P
       [60]},{P[60],P[53]},{P[53],P[45]},{P[45],P[41]},{P[41],P[22]},{P[22],P[20]},{P[20],P
       [36]}};
4  Poligono3={{P[33],P[43]},{P[43],P[56]},{P[56],P[50]},{P[50],P[37]},{P[37],P[39]},{P[39],P
       [26]},{P[26],P[14]},{P[14],P[21]},{P[21],P[8]},{P[8],P[15]},{P[15],P[30]},{P[30],P
       [33]}};
5  Poligono4={{P[44],P[49]},{P[49],P[58]},{P[58],P[59]},{P[59],P[51]},{P[51],P[52]},{P[52],P
       [38]},{P[38],P[29]},{P[29],P[40]},{P[40],P[23]},{P[23],P[18]},{P[18],P[35]},{P[35],P
       [44]}};
6  PC[1]=RegionCentroid[ConvexHullRegion[Flatten[Poligono1,1]]];
7  PC[2]=RegionCentroid[ConvexHullRegion[Flatten[Poligono2,1]]];
8  PC[3]=RegionCentroid[ConvexHullRegion[Flatten[Poligono3,1]]];
9  PC[4]=RegionCentroid[ConvexHullRegion[Flatten[Poligono4,1]]];
10
11 {ArcCos[PC[1].PC[2]],ArcCos[PC[1].PC[3]],ArcCos[PC[1].PC[4]],ArcCos[PC[2].PC[3]],ArcCos[PC
       [2].PC[4]],ArcCos[PC[3].PC[4]]}
12
13 Show[Graphics3D[{Yellow,Sphere[],Black,Table[Inset[Text[Style[i,20]],(1.02)P[i]],{i
       ,1,60}]}],ParametricPlot3D[Table[Arc[Part[AristasCortas,i],t],{i,1,119}],{t,0,1},
       PlotStyle->Red],ParametricPlot3D[Table[Arc[Part[AristasSemicortas,i],t],{i,1,19}],{t
       ,0,1},PlotStyle->Yellow],ParametricPlot3D[Table[Arc[Part[AristasMedianas,i],t],{i
       ,1,24}],{t,0,1},PlotStyle->Blue],ParametricPlot3D[Table[Arc[Part[AristasLargas,i],t],{i
       ,1,12}],{t,0,1},PlotStyle->Green],ParametricPlot3D[Table[Arc[Part[Poligono1,i],t],{i
       ,1,12}],{t,0,1},PlotStyle->{LightGray,Thickness[.02]}],ParametricPlot3D[Table[Arc[Part[
       Poligono2,i],t],{i,1,12}],{t,0,1},PlotStyle->{LightGray,Thickness[.02]}],
       ParametricPlot3D[Table[Arc[Part[Poligono3,i],t],{i,1,12}],{t,0,1},PlotStyle->{LightGray,
       Thickness[.02]}],ParametricPlot3D[Table[Arc[Part[Poligono4,i],t],{i,1,12}],{t,0,1},
       PlotStyle->{LightGray,Thickness[.02]}],ParametricPlot3D[Arc[{PC[1],PC[2]},t],{t,0,1},
       PlotStyle->Black],ParametricPlot3D[Arc[{PC[1],PC[3]},t],{t,0,1},PlotStyle->Black],
```

```
    ParametricPlot3D[Arc[{PC[1],PC[4]},t],{t,0,1},PlotStyle->Black],ParametricPlot3D[Arc[{PC
    [2],PC[3]},t],{t,0,1},PlotStyle->Black],ParametricPlot3D[Arc[{PC[2],PC[4]},t],{t,0,1},
    PlotStyle->Black],ParametricPlot3D[Arc[{PC[3],PC[4]},t],{t,0,1},PlotStyle->Black]]
Out[573]= {1.73178,1.72475,1.72901,1.72901,1.72475,1.72672}
Out[574]=
```

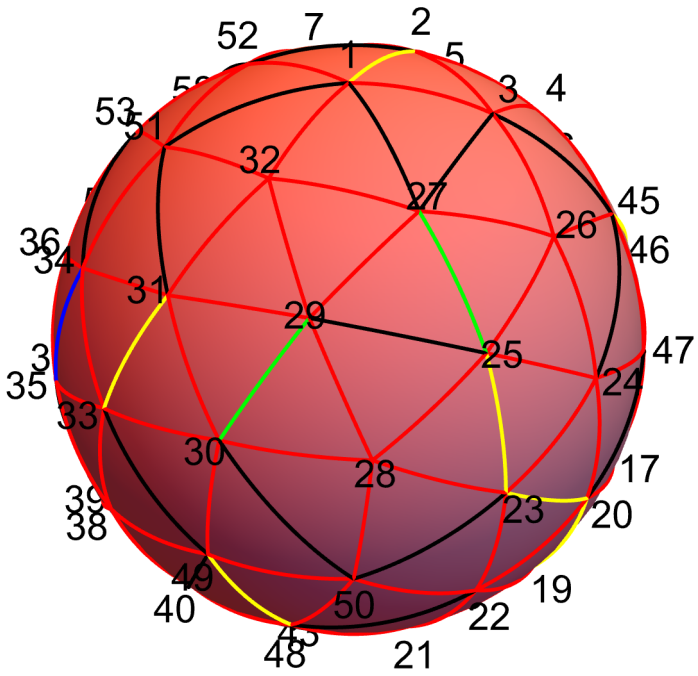


```

In[2]:= SetPrecision[\[Alpha],9]-SetPrecision[\[Alpha]0,9]
SetPrecision[ArcCos[\[Alpha]0],10]
SetPrecision[ArcCos[\[Alpha]],10]
2*10^(-9)+3*10^(-10)+6*10^(-11)+8*10^(-12.)
Out[2]= \[Alpha]-\[Alpha]0
Out[3]= ArcCos[\[Alpha]0]
Out[4]= ArcCos[\[Alpha]]
Out[5]= 2.368*10^-9
In[6]:= 1.07628177722176582531378105644925824436`10.1
    -1.07628177133472242588954279063351293644`10.
Out[6]= 6.*10^-9
In[138]:= \[Alpha]0=ArcCos[0.889473568664781383`8.688124857428225]
{\[Epsilon]c\[Alpha]=-2/10^9-3/10^10-6/10^11-8/10^12-9/10^13,(*\[Epsilon]c\[Beta
    ]1=-1/10^8-8/10^9-5/10^10-3/10^11-9/10^13,\[Epsilon]c\[Beta]2=0,\[Epsilon]c\[Beta
    ]3=-4/10^13,\[Epsilon]c\[Beta]4=-4/10^9-1/10^11-9/10^12-8/10^13,\[Epsilon]c\[Gamma
    ]1=0,\[Epsilon]c\[Gamma]2=0,\[Epsilon]c\[Gamma]3=0,\[Epsilon]c\[Gamma]4=0,\[Epsilon]c\[
    Delta]=0*)};
\[Epsilon]c\[Alpha]=2*10^(-9)+3*10^(-10)+6*10^(-11)+8*10^(-12);
\[Alpha]=ArcCos[-\[Epsilon]c\[Alpha]+Cos[\[Alpha]0]];
A=ArcCos[Cos[\[Alpha]]/(1+Cos[\[Alpha]])];
{\[Beta]1=ArcCos[0.8893185854087823867],\[Beta]2=ArcCos[0.8892732945383421843],\[Beta]3=
    ArcCos[0.8891003676635580072],\[Beta]4=ArcCos[0.8887814713774418999]};
{B1=ArcCos[(Cos[\[Beta]1]-Cos[\[Alpha]]^2)/Sin[\[Alpha]]^2],B2=ArcCos[(Cos[\[Beta]2]-Cos[\[
    Alpha]]^2)/Sin[\[Alpha]]^2],B3=ArcCos[(Cos[\[Beta]3]-Cos[\[Alpha]]^2)/Sin[\[Alpha]]^2],
    B4=ArcCos[(Cos[\[Beta]4]-Cos[\[Alpha]]^2)/Sin[\[Alpha]]^2]};
{\[Gamma]1=ArcCos[0.8880460352643052995],\[Gamma]2=ArcCos[0.8870695855434101028],\[Gamma]3=
```

```
    ArcCos[0.8864807165749388689],\[Gamma]4=ArcCos[0.8862670508008343837]};
{G1=ArcCos[(Cos[\[Gamma]1]-Cos[\[Alpha]]^2)/Sin[\[Alpha]]^2],G2=ArcCos[(Cos[\[Gamma]2]-Cos
    [\[Alpha]]^2)/Sin[\[Alpha]]^2],G3=ArcCos[(Cos[\[Gamma]3]-Cos[\[Alpha]]^2)/Sin[\[Alpha
    ]]^2],
G4=ArcCos[(Cos[\[Gamma]4]-Cos[\[Alpha]]^2)/Sin[\[Alpha]]^2]};
\[Delta]=ArcCos[0.8821680683605515049];
De=ArcCos[(Cos[\[Delta]]-Cos[\[Alpha]]^2)/Sin[\[Alpha]]^2];
Rz[\[Theta]_]:=(Cos[\[Theta]] -Sin[\[Theta]]  0
Sin[\[Theta]] Cos[\[Theta]] 0
0 0 1

);
Rx=(-1  0 0
0 1 0
0 0 1

);
Arc[A_,B_,t_]:=((1-t)A+t B)/Norm[((1-t)A+t B)]
Refl[x_,y_]:=ReflectionMatrix[Cross[x,y]]
Rot[x_,y_]:=Transpose[{Cross[y,x]/Norm[Cross[x,y]],(y-y.x x)/Norm[(y-y.x x)],x}]
T[1]={0,0,1};
T[2]={0,Sin[\[Alpha]],Cos[\[Alpha]]};
T[3]=Rz[-A].T[2];
Q[1]={0,0,1};
Q[2]={0,Sin[\[Beta]2],Cos[\[Beta]2]};
Q[3]={x,y,z}/.FindRoot[Q[1].{x,y,z}==Cos[\[Alpha]]&&Q[2].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z
    }]==1,{{x,0.40443374669294246},{y,0.2127677493950803},{z,0.8894735686647813}}];
Q[4]=Rot[Q[3],Q[2]].T[3];
Q[4]=Rot[Q[3],Q[2]].T[3];
Q[5]={x,y,z}/.FindRoot[Q[2].{x,y,z}==Cos[\[Alpha]]&&Q[4].{x,y,z}==Cos[\[Gamma]3]&&Norm[{x,y,
    z}]==1,{{x,-0.048902394720918146},{y,0.810876540954435},{z,0.5831704648902669}}];
Q[6]={x,y,z}/.FindRoot[Q[4].{x,y,z}==Cos[\[Alpha]]&&Q[5].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z
    }]==1,{{x,0.3081940854511294},{y,0.9037453729090233},{z,0.2970870354601019}}];
Q[7]=Rot[Q[5],Q[2]].T[3];
Q[8]={x,y,z}/.FindRoot[Q[7].{x,y,z}==Cos[\[Alpha]]&&Q[5].{x,y,z}==Cos[\[Gamma]2]&&Norm[{x,y,
    z}]==1,{{x,-0.4555950217704665},{y,0.8261716393795384},{z,0.3314718667140452}}];
Q[9]={x,y,z}/.FindRoot[Q[6].{x,y,z}==Cos[\[Alpha]]&&Q[8].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z
    }]==1,{{x,-0.09734895146906396},{y,0.9923204372889003},{z,0.07631075537982657}}];
Q[10]=Rot[Q[6],Q[9]].T[3];
Q[11]=Rot[Q[9],Q[8]].T[3];
Q[12]={x,y,z}/.FindRoot[Q[10].{x,y,z}==Cos[\[Alpha]]&&Q[11].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,
    y,z}]==1,{{x,-0.11041521747336673},{y,0.9090597828639854},{z,-0.4017695744199567}}];
Q[13]={x,y,z}/.FindRoot[Q[12].{x,y,z}==Cos[\[Alpha]]&&Q[10].{x,y,z}==Cos[\[Beta]1]&&Norm[{x,
    y,z}]==1,{{x,0.2834468762504213},{y,0.7472186148824155},{z,-0.6011008317388046}}];
Q[14]={x,y,z}/.FindRoot[Q[10].{x,y,z}==Cos[\[Alpha]]&&Q[13].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,
    y,z}]==1,{{x,0.6583718228692461},{y,0.6785039712462315},{z,-0.325851045502265}}];
Q[15]={x,y,z}/.FindRoot[Q[13].{x,y,z}==Cos[\[Alpha]]&&Q[14].{x,y,z}==Cos[\[Gamma]4]&&Norm[{x
    ,y,z}]==1,{{x,0.5834596726375917},{y,0.4008475640272482},{z,-0.7063257328025472}}];
Q[16]=Rot[Q[15],Q[13]].T[3];
Q[17]={x,y,z}/.FindRoot[Q[14].{x,y,z}==Cos[\[Alpha]]&&Q[15].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,
    y,z}]==1,{{x,0.8877301228068757},{y,0.2707227858496931},{z,-0.3723498385699538}}];
Q[18]=Rot[Q[15],Q[16]].T[3];
Q[19]={x,y,z}/.FindRoot[Q[17].{x,y,z}==Cos[\[Alpha]]&&Q[18].{x,y,z}==Cos[\[Gamma]1]&&Norm[{x
    ,y,z}]==1,{{x,0.7711273309577059},{y,-0.10642600147777967},{z,-0.6277229848105742}}];
Q[20]={x,y,z}/.FindRoot[Q[17].{x,y,z}==Cos[\[Alpha]]&&Q[19].{x,y,z}==Cos[\[Beta]4]&&Norm[{x,
    y,z}]==1,{{x,0.9662341298966517},{y,-0.1602988130216733},{z,-0.2017322402758248}}];
Q[21]={x,y,z}/.FindRoot[Q[18].{x,y,z}==Cos[\[Alpha]]&&Q[19].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,
    y,z}]==1,{{x,0.464748763786155},{y,-0.4322214941612015},{z,-0.7727827421366883}}];
Q[22]=Rot[Q[19],Q[21]].T[3];
Q[23]={x,y,z}/.FindRoot[Q[22].{x,y,z}==Cos[\[Alpha]]&&Q[20].{x,y,z}==Cos[\[Beta]3]&&Norm[{x,
    y,z}]==1,{{x,0.8418337464568276},{y,-0.5372551413485798},{z,0.05169967525023173}}];
Q[24]={x,y,z}/.FindRoot[Q[20].{x,y,z}==Cos[\[Alpha]]&&Q[23].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,
    y,z}]==1,{{x,0.9540374286644094},{y,0.13494235048096523},{z,0.2675876431266477}}];
Q[25]={x,y,z}/.FindRoot[Q[24].{x,y,z}==Cos[\[Alpha]]&&Q[23].{x,y,z}==Cos[\[Beta]4]&&Norm[{x,
    y,z}]==1,{{x,0.7245097631148443},{y,-0.470591925476885},{z,0.5036157690414689}}];
Q[26]=Rot[Q[24],Q[25]].T[3];
Q[27]={x,y,z}/.FindRoot[Q[26].{x,y,z}==Cos[\[Alpha]]&&Q[25].{x,y,z}==Cos[\[Gamma]1]&&Norm[{x
    ,y,z}]==1,{{x,0.42235469906842704},{y,-0.33583457084856505},{z,0.84192140321872}}];
```

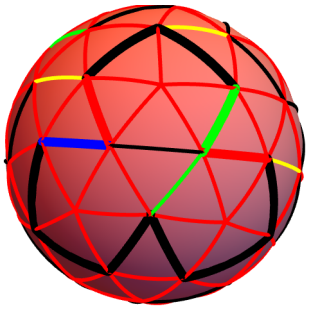
```
Q[28]={x,y,z}/.FindRoot[Q[23].{x,y,z}==Cos[\[Alpha]]&&Q[25].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,0.5212218030958334},{y,-0.8142975068370093},{z,0.2554357107699687}}];
Q[29]={x,y,z}/.FindRoot[Q[27].{x,y,z}==Cos[\[Alpha]]&&Q[28].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,0.2600312007223925},{y,-0.726266982962316},{z,0.6363332806868474}}];
Q[30]={x,y,z}/.FindRoot[Q[28].{x,y,z}==Cos[\[Alpha]]&&Q[29].{x,y,z}==Cos[\[Gamma]4]&&Norm[{x,y,z}]==1,{{x,0.07474790554042843},{y,-0.9613421183385972},{z,0.26501713553198375}}];Q[31]={x,y,z}/.FindRoot[Q[29].{x,y,z}==Cos[\[Alpha]]&&Q[30].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.2054900871274366},{y,-0.777294403977514},{z,0.594632015315018}}];
Q[32]=Rot[Q[29],Q[31]].T[3];Q[33]={x,y,z}/.FindRoot[Q[30].{x,y,z}==Cos[\[Alpha]]&&Q[31].{x,y,z}==Cos[\[Beta]1]&&Norm[{x,y,z}]==1,{{x,-0.3840889961505356},{y,-0.9058248379226465},{z,0.17876522603259656}}];
Q[34]={x,y,z}/.FindRoot[Q[31].{x,y,z}==Cos[\[Alpha]]&&Q[33].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.6298845725519405},{y,-0.6234515635140365},{z,0.463199280237958}}];
Q[35]={x,y,z}/.FindRoot[Q[33].{x,y,z}==Cos[\[Alpha]]&&Q[34].{x,y,z}==Cos[\[Delta]]&&Norm[{x,y,z}]==1,{{x,-0.744577923672671},{y,-0.6675044103325863},{z,-0.006448082338236139 6}}];
Q[36]={x,y,z}/.FindRoot[Q[34].{x,y,z}==Cos[\[Alpha]]&&Q[35].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.9149959071202968},{y,-0.3143331664012362},{z,0.25293704839994824}}];
Q[37]=Rot[Q[36],Q[35]].T[3];
Q[38]=Rot[Q[35],Q[33]].T[3];
Q[39]={x,y,z}/.FindRoot[Q[38].{x,y,z}==Cos[\[Alpha]]&&Q[37].{x,y,z}==Cos[\[Gamma]2]&&Norm[{x,y,z}]==1,{{x,-0.6635200981536724},{y,-0.5340800651215074},{z,-0.5239270592228916}}];
Q[40]={x,y,z}/.FindRoot[Q[38].{x,y,z}==Cos[\[Alpha]]&&Q[39].{x,y,z}==Cos[\[Gamma]3]&&Norm[{x,y,z}]==1,{{x,-0.24204679638935747},{y,-0.6860388999293006},{z,-0.686122420666635 8}}];
Q[41]={x,y,z}/.FindRoot[Q[40].{x,y,z}==Cos[\[Alpha]]&&Q[39].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.4330561956675092},{y,-0.2903028108925441},{z,-0.853338508085667}}];
Q[42]=Rot[Q[39],Q[41]].T[3];
Q[43]=Rot[Q[41],Q[40]].T[3];
Q[44]={x,y,z}/.FindRoot[Q[43].{x,y,z}==Cos[\[Alpha]]&&Q[41].{x,y,z}==Cos[\[Beta]2]&&Norm[{x,y,z}]==1,{{x,-0.14359695911048806},{y,0.055399817080754414},{z,-0.9880843959913749}}];
Q[45]={x,y,z}/.FindRoot[Q[26].{x,y,z}==Cos[\[Alpha]]&&Q[4].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,0.7579151060535679},{y,0.4080571968885414},{z,0.5089734925153518}}];
Q[46]={x,y,z}/.FindRoot[Q[6].{x,y,z}==Cos[\[Alpha]]&&Q[14].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,0.7015949106533736},{y,0.6983040187998999},{z,0.14190165141108838}}];
Q[47]={x,y,z}/.FindRoot[Q[17].{x,y,z}==Cos[\[Alpha]]&&Q[24].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,0.9489427036305138},{y,0.3014911931657835},{z,0.09279442693331792}}];
Q[48]={x,y,z}/.FindRoot[Q[21].{x,y,z}==Cos[\[Alpha]]&&Q[40].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,0.2081461962717046},{y,-0.7807643815405613},{z,-0.5891367765597442}}];
Q[49]={x,y,z}/.FindRoot[Q[30].{x,y,z}==Cos[\[Alpha]]&&Q[38].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,0.02593583349519103},{y,-0.9789859321890553},{z,-0.2022717902151371}}];
Q[50]={x,y,z}/.FindRoot[Q[22].{x,y,z}==Cos[\[Alpha]]&&Q[28].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,0.47860298591121175},{y,-0.8522394055135843},{z,-0.2112514557742139}}];Q[51]={x,y,z}/.FindRoot[Q[32].{x,y,z}==Cos[\[Alpha]]&&Q[34].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.4786754662045244},{y,-0.34241957281472607},{z,0.8084668417487894}}];Q[52]={x,y,z}/.FindRoot[Q[1].{x,y,z}==Cos[\[Alpha]]&&Q[7].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.4411692849625412},{y,0.11919074063189028},{z,0.8894735686647813}}];
Q[53]={x,y,z}/.FindRoot[Q[51].{x,y,z}==Cos[\[Alpha]]&&Q[52].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.7294466145870048},{y,-0.174047454315023},{z,0.6615250652035971}}];
Q[54]={x,y,z}/.FindRoot[Q[42].{x,y,z}==Cos[\[Alpha]]&&Q[44].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.5295804439302658},{y,0.25532524296545095},{z,-0.8089212407343844}}];
Q[55]={x,y,z}/.FindRoot[Q[16].{x,y,z}==Cos[\[Alpha]]&&Q[12].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.2371607412198595},{y,0.6185897230073244},{z,-0.7490671114217794}}];
Q[56]={x,y,z}/.FindRoot[Q[54].{x,y,z}==Cos[\[Alpha]]&&Q[55].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.6336837840139935},{y,0.592126667243545},{z,0.49782614617631193}}];
Q[58]={x,y,z}/.FindRoot[Q[53].{x,y,z}==Cos[\[Alpha]]&&Q[7].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.7770574165230251},{y,0.28303198713070393},{z,0.5622051811282133}}];
Q[60]={x,y,z}/.FindRoot[Q[42].{x,y,z}==Cos[\[Alpha]]&&Q[56].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.91156030907628},{y,0.2563762021926282},{z,-0.3214483564525433}}];
Q[57]={x,y,z}/.FindRoot[Q[58].{x,y,z}==Cos[\[Alpha]]&&Q[60].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.9187422383471895},{y,0.14134119579867604},{z,0.3686941358999306}}];
Q[59]={x,y,z}/.FindRoot[Q[58].{x,y,z}==Cos[\[Alpha]]&&Q[60].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.7975241230868134},{y,0.5935818022969545},{z,0.10777716398434424}}];
Malas={};
For[i=1,i<60,i++,For[j=i+1,j<=60,j++,If[Q[i].Q[j]>Cos[\[Alpha]],Malas=Union[Malas,{{i,j,Q[i].Q[j]-Cos[\[Alpha]]}}]]]]
Print["Malas = ",Malas]
A\[Alpha]={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Cos[\[Alpha]]-10^(-8)<=Q[i].Q[j]<=Cos[\[Alpha]],A\[Alpha]=Union[A\[Alpha] ,{{i,j}}]]]]
```

```
A\[Beta]={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Cos[\[Beta]4]<=Q[i].Q[j]<=Cos[\[Beta]1],A\[Beta]=
    Union[A\[Beta] ,{{i,j}}]]]]
A\[Gamma]={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Cos[\[Gamma]4]<=Q[i].Q[j]<=Cos[\[Gamma]1],A\[Gamma
    ]=Union[A\[Gamma] ,{{i,j}}]]]]
A\[Delta]={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Cos[\[Delta]]<=Q[i].Q[j]<=Cos[\[Delta]]+10^(-8),A\[
    Delta]=Union[A\[Delta] ,{{i,j}}]]]]
AL={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[0.79<Q[i].Q[j]<Cos[\[Delta]],AL=Union[AL ,{{i,j
    }}]]]]
Print["A\[Alpha] = ",Length[A\[Alpha]],"   A\[Beta] = ",Length[A\[Beta]],"   A\[Gamma] = ",
    Length[A\[Gamma]],"   A\[Delta] = ",Length[A\[Delta]],"   AL = ",Length[AL]]
Poligono1={{Q[35],Q[33]},{Q[33],Q[49]},{Q[49],Q[40]},{Q[40],Q[39]},{Q[39],Q[41]},{Q[41],Q
    [54]},{Q[54],Q[60]},{Q[60],Q[37]},{Q[37],Q[57]},{Q[57],Q[53]},{Q[53],Q[34]},{Q[34],Q
    [35]}};
Poligono2={{Q[8],Q[59]},{Q[59],Q[56]},{Q[56],Q[12]},{Q[12],Q[9]},{Q[9],Q[10]},{Q[10],Q
    [46]},{Q[46],Q[4]},{Q[4],Q[5]},{Q[5],Q[2]},{Q[2],Q[52]},{Q[52],Q[58]},{Q[58],Q[8]}};
Poligono3={{Q[27],Q[3]},{Q[3],Q[45]},{Q[45],Q[24]},{Q[24],Q[25]},{Q[25],Q[23]},{Q[23],Q
    [50]},{Q[50],Q[30]},{Q[30],Q[29]},{Q[29],Q[31]},{Q[31],Q[51]},{Q[51],Q[1]},{Q[1],Q
    [27]}};
Poligono4={{Q[19],Q[20]},{Q[20],Q[47]},{Q[47],Q[14]},{Q[14],Q[15]},{Q[15],Q[13]},{Q[13],Q
    [55]},{Q[55],Q[44]},{Q[44],Q[18]},{Q[18],Q[43]},{Q[43],Q[48]},{Q[48],Q[22]},{Q[22],Q
    [19]}};
PC[1]=RegionCentroid[ConvexHullRegion[Flatten[Poligono1,1]]];
PC[2]=RegionCentroid[ConvexHullRegion[Flatten[Poligono2,1]]];
PC[3]=RegionCentroid[ConvexHullRegion[Flatten[Poligono3,1]]];
PC[4]=RegionCentroid[ConvexHullRegion[Flatten[Poligono4,1]]];
{ArcCos[PC[1].PC[2]],ArcCos[PC[1].PC[3]],ArcCos[PC[1].PC[4]],ArcCos[PC[2].PC[3]],ArcCos[PC
    [2].PC[4]],ArcCos[PC[3].PC[4]]};

Show[Graphics3D[{Opacity[1],Pink,Sphere[],Opacity[1],Black,Table[Inset[Text[Style[i
    ,20]],(1.1)Q[i]],{i,1,60}]},Boxed->False],Table[ParametricPlot3D[Arc[Q[Part[A\[Alpha],i
    ,1]],Q[Part[A\[Alpha],i,2]],t],{t,0,1},PlotStyle->Red],{i,1,Length[A\[Alpha]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Beta],i,1]],Q[Part[A\[Beta],i,2]],t],{t,0,1},PlotStyle
    ->Yellow],{i,1,Length[A\[Beta]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Gamma],i,1]],Q[Part[A\[Gamma],i,2]],t],{t,0,1},
    PlotStyle->Green],{i,1,Length[A\[Gamma]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Delta],i,1]],Q[Part[A\[Delta],i,2]],t],{t,0,1},
    PlotStyle->Blue],{i,1,Length[A\[Delta]]}],
Table[ParametricPlot3D[Arc[Q[Part[AL,i,1]],Q[Part[AL,i,2]],t],{t,0,1},PlotStyle->Black],{i
    ,1,Length[AL]}],ParametricPlot3D[Table[Arc[Part[Poligono1,i],t],{i,1,12}],{t,0,1},
    PlotStyle->{LightGray,Thickness[.01]}],ParametricPlot3D[Table[Arc[Part[Poligono2,i],t],{
    i,1,12}],{t,0,1},PlotStyle->{LightGray,Thickness[.01]}],ParametricPlot3D[Table[Arc[Part[
    Poligono3,i],t],{i,1,12}],{t,0,1},PlotStyle->{LightGray,Thickness[.01]}],
    ParametricPlot3D[Table[Arc[Part[Poligono4,i],t],{i,1,12}],{t,0,1},PlotStyle->{LightGray,
    Thickness[.01]}],ParametricPlot3D[Arc[{PC[1],PC[2]},t],{t,0,1},PlotStyle->Black],
    ParametricPlot3D[Arc[{PC[1],PC[3]},t],{t,0,1},PlotStyle->Black],ParametricPlot3D[Arc[{PC
    [1],PC[4]},t],{t,0,1},PlotStyle->Black],ParametricPlot3D[Arc[{PC[2],PC[3]},t],{t,0,1},
    PlotStyle->Black],ParametricPlot3D[Arc[{PC[2],PC[4]},t],{t,0,1},PlotStyle->Black],
    ParametricPlot3D[Arc[{PC[3],PC[4]},t],{t,0,1},PlotStyle->Black]]
Out[138]= 0.47460441
Durante la evaluaci\363n de In[138]:= Malas = {}
Durante la evaluaci\363n de In[138]:= A\[Alpha] = 119   A\[Beta] = 9   A\[Gamma] = 8   A\[
    Delta] = 2   AL = 36
Out[237]=
```

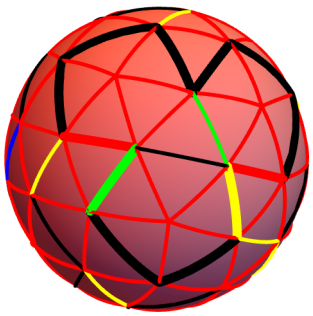



```
In[107]:= A\[Alpha]={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Cos[\[Alpha]]-10^(-8)<=Q[i].Q[j]<=Cos[\[Alpha]],A\[
    Alpha]=Union[A\[Alpha],{{i,j}}]]]]
A\[Beta]1={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[ Q[i].Q[j]==Cos[\[Beta]1],A\[Beta]1=Union[A\[Beta]1
    ,{{i,j}}]]]]
A\[Beta]2={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[ Q[i].Q[j]==Cos[\[Beta]2],A\[Beta]2=Union[A\[Beta]2
    ,{{i,j}}]]]]
A\[Beta]3={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Cos[\[Beta]3]-3*10^(-4)<Q[i].Q[j]<=Cos[\[Beta]3],A
    \[Beta]3=Union[A\[Beta]3 ,{{i,j}}]]]]
A\[Beta]4={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[ Q[i].Q[j]==Cos[\[Beta]4],A\[Beta]4=Union[A\[Beta]4
    ,{{i,j}}]]]]
A\[Gamma]1={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Q[i].Q[j]==Cos[\[Gamma]1],A\[Gamma]1=Union[A\[Gamma
    ]1 ,{{i,j}}]]]]
A\[Gamma]2={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Q[i].Q[j]==Cos[\[Gamma]2],A\[Gamma]2=Union[A\[Gamma
    ]2 ,{{i,j}}]]]]
A\[Gamma]3={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Q[i].Q[j]==Cos[\[Gamma]3],A\[Gamma]3=Union[A\[Gamma
    ]3 ,{{i,j}}]]]]
A\[Gamma]4={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Q[i].Q[j]==Cos[\[Gamma]4],A\[Gamma]4=Union[A\[Gamma
    ]4 ,{{i,j}}]]]]
A\[Delta]={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Cos[\[Delta]]<=Q[i].Q[j]<=Cos[\[Delta]]+10^(-8),A\[
    Delta]=Union[A\[Delta] ,{{i,j}}]]]]
AL={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[0.79<Q[i].Q[j]<Cos[\[Delta]],AL=Union[AL ,{{i,j
    }}]]]]
AT={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[0.79<Q[i].Q[j]<=Cos[\[Alpha]],AT=Union[AT ,{{i,j
    }}]]]]

Print["A\[Alpha] = ",Length[A\[Alpha]],"   A\[Beta]1 = ",Length[A\[Beta]1],"   A\[Beta]2 = "
```

```
    ,Length[A\[Beta]2],"   A\[Beta]3 = ",Length[A\[Beta]3],"   A\[Beta]4 = ",Length[A\[Beta
    ]4],"   A\[Gamma]1 = ",Length[A\[Gamma]1],"   A\[Gamma]2 = ",Length[A\[Gamma]2],"   A\[
    Gamma]3 = ",Length[A\[Gamma]3],"   A\[Gamma]4 = ",Length[A\[Gamma]4],"   A\[Delta] = ",
    Length[A\[Delta]],"   AL = ",Length[AL],"   AT = ",Length[AT]]
Durante la evaluaci\363n de In[107]:= A\[Alpha] = 119   A\[Beta]1 = 2   A\[Beta]2 = 2   A\[
    Beta]3 = 3   A\[Beta]4 = 2   A\[Gamma]1 = 2   A\[Gamma]2 = 2   A\[Gamma]3 = 2   A\[Gamma
    ]4 = 2   A\[Delta] = 2   AL = 36   AT = 174
In[132]:= Show[Graphics3D[{Opacity[1],Pink,Sphere[]},Boxed->False],Table[ParametricPlot3D[
    Arc[Q[Part[A\[Alpha],i,1]],Q[Part[A\[Alpha],i,2]],t],{t,0,1},PlotStyle->Red],{i,1,Length
    [A\[Alpha]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Beta],i,1]],Q[Part[A\[Beta],i,2]],t],{t,0,1},PlotStyle
    ->Yellow],{i,1,Length[A\[Beta]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Gamma],i,1]],Q[Part[A\[Gamma],i,2]],t],{t,0,1},
    PlotStyle->Green],{i,1,Length[A\[Gamma]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Delta],i,1]],Q[Part[A\[Delta],i,2]],t],{t,0,1},
    PlotStyle->Blue],{i,1,Length[A\[Delta]]}],
Table[ParametricPlot3D[Arc[Q[Part[AL,i,1]],Q[Part[AL,i,2]],t],{t,0,1},PlotStyle->Black],{i
    ,1,Length[AL]}],ParametricPlot3D[Arc[Q[37],Q[57],t],{t,0,1},PlotStyle->{Thickness[.03],
    Black}],ParametricPlot3D[Arc[Q[57],Q[53],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[53],Q[34],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[34],Q[35],t],{t,0,1},PlotStyle->{Thickness[.03],Blue}],
    ParametricPlot3D[Arc[Q[35],Q[33],t],{t,0,1},PlotStyle->{Thickness[.03],Red}],
    ParametricPlot3D[Arc[Q[33],Q[49],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[49],Q[40],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[40],Q[39],t],{t,0,1},PlotStyle->{Thickness[.03],Green}],
    ParametricPlot3D[Arc[Q[39],Q[41],t],{t,0,1},PlotStyle->{Thickness[.03],Red}],
    ParametricPlot3D[Arc[Q[41],Q[54],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[54],Q[60],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[60],Q[37],t],{t,0,1},PlotStyle->{Thickness[.03],Black}]]
```



```

Show[Graphics3D[{Opacity[1],Pink,Sphere[]},Boxed->False],Table[ParametricPlot3D[Arc[Q[Part[A
    \[Alpha],i,1]],Q[Part[A\[Alpha],i,2]],t],{t,0,1},PlotStyle->Red],{i,1,Length[A\[Alpha
    ]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Beta],i,1]],Q[Part[A\[Beta],i,2]],t],{t,0,1},PlotStyle
    ->Yellow],{i,1,Length[A\[Beta]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Gamma],i,1]],Q[Part[A\[Gamma],i,2]],t],{t,0,1},
    PlotStyle->Green],{i,1,Length[A\[Gamma]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Delta],i,1]],Q[Part[A\[Delta],i,2]],t],{t,0,1},
    PlotStyle->Blue],{i,1,Length[A\[Delta]]}],
Table[ParametricPlot3D[Arc[Q[Part[AL,i,1]],Q[Part[AL,i,2]],t],{t,0,1},PlotStyle->Black],{i
    ,1,Length[AL]}],ParametricPlot3D[Arc[Q[23],Q[50],t],{t,0,1},PlotStyle->{Thickness[.03],
    Black}],ParametricPlot3D[Arc[Q[50],Q[30],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[30],Q[29],t],{t,0,1},PlotStyle->{Thickness[.03],Green}],
    ParametricPlot3D[Arc[Q[29],Q[31],t],{t,0,1},PlotStyle->{Thickness[.03],Red}],
    ParametricPlot3D[Arc[Q[31],Q[51],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[51],Q[1],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[1],Q[27],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[27],Q[3],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[3],Q[45],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[45],Q[24],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[24],Q[25],t],{t,0,1},PlotStyle->{Thickness[.03],Red}],
    ParametricPlot3D[Arc[Q[25],Q[23],t],{t,0,1},PlotStyle->{Thickness[.03],Yellow}]]
Out[133]=
```

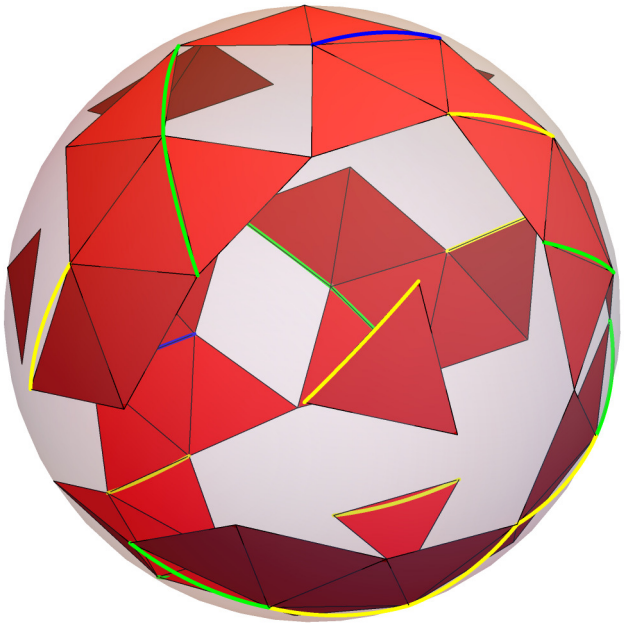


```
In[134]:= Grid[Table[{"Q",i,SetPrecision[Part[Q[i],1],9],SetPrecision[Part[Q[i],2],9],
    SetPrecision[Part[Q[i],3],9]},{i,1,60}]];
In[135]:= (* Tri ngulos m nimos *)
Y={};
For[i=1,i<=60,i++,
For[j=i+1,j<=60,j++,For[k=j+1,k<=60,k++,If[MemberQ[A\[Alpha],{i,j}]&&MemberQ[A\[Alpha],{i,k
    }]&&MemberQ[A\[Alpha],{j,k}],Y=Union[Y,{{i,j,k}}]]]]]
Print["Length[Y]= ",Length[Y]]
Durante la evaluaci\363n de In[135]:= Length[Y]= 18
```

Appendix 1 - Wolfram's program Code

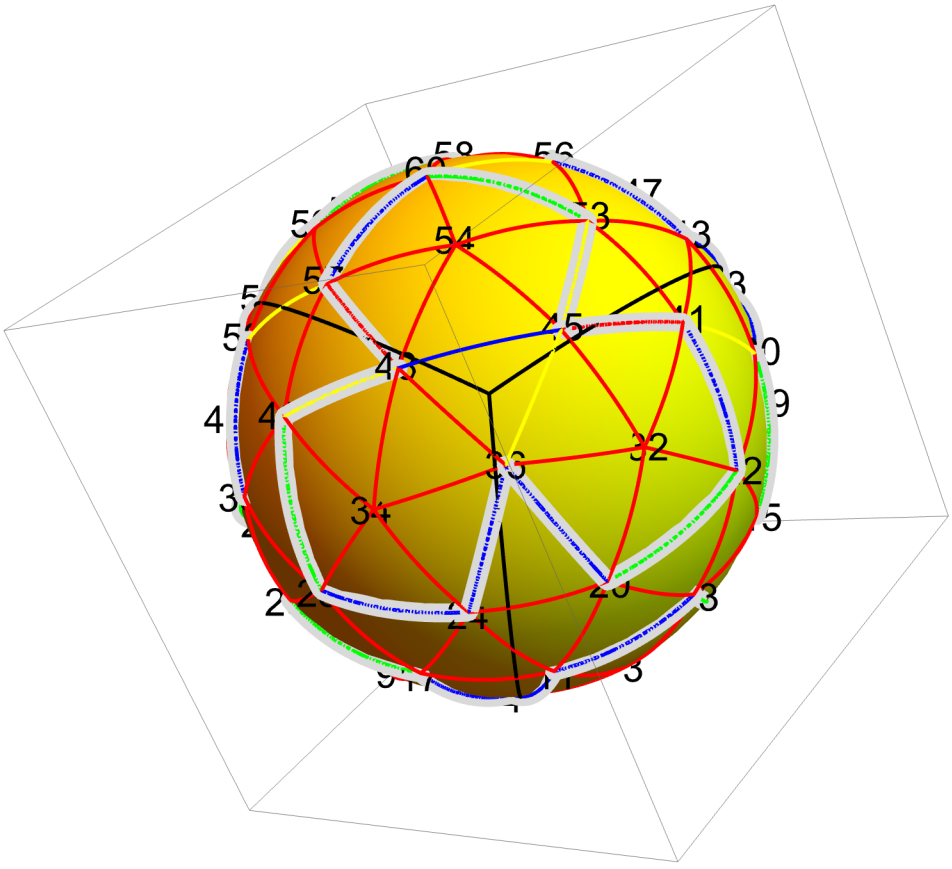
```
Puntos=Flatten[Import["D:\\A\\Anahuac\\Investigaci n y anexas\\Puntos en la esfera\\60
    num rico\\60 num rico.xlsx"],1];
For[i=1,i<=60,i++,P[i]=SetPrecision[Part[Puntos,i],9]]
Pu=Table[P[i],{i,1,60}];

AristasCortas={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[.889473<P[i].P[j],AristasCortas=Union[AristasCortas
    ,{{P[i],P[j]}}]]]]
AristasSemicortas={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[.882168<P[i].P[j]<.889319,AristasSemicortas=Union[
    AristasSemicortas,{{P[i],P[j]}}]]]]
AristasMedianas={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[.840668<P[i].P[j]<.853922,AristasMedianas=Union[
    AristasMedianas,{{P[i],P[j]}}]]]]
AristasLargas={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[.799082<P[i].P[j]<.808467,AristasLargas=Union[
    AristasLargas,{{P[i],P[j]}}]]]]
{Length[AristasCortas],Length[AristasSemicortas],Length[AristasMedianas],Length[
    AristasLargas],Length[AristasCortas]+Length[AristasSemicortas]+Length[AristasMedianas]+
    Length[AristasLargas]}
Out[550]= {119,19,24,12,174}

X=Union[AristasCortas,AristasSemicortas];
Y={};
For[i=1,i<=60,i++,
For[j=i+1,j<=60,j++,For[k=j+1,k<=60,k++,If[MemberQ[X,{P[i],P[j]}]&&MemberQ[X,{P[i],P[k]}]&&
    MemberQ[X,{P[j],P[k]}],Y=Union[Y,{{P[i],P[j],P[k]}}]]]]]
Print["Length[Y]= ",Length[Y]]

AristasSemicortas1={};
AristasSemicortas2={};
AristasSemicortas3={};

For[i=1,i<=19,i++,If[0.8886<Part[AristasSemicortas,i,1].Part[AristasSemicortas,i,2],
    AristasSemicortas1=Union[AristasSemicortas1,{Part[AristasSemicortas,i]}],If[0.886<Part[
    AristasSemicortas,i,1].Part[AristasSemicortas,i,2],AristasSemicortas2=Union[
    AristasSemicortas2,{Part[AristasSemicortas,i]}],AristasSemicortas3=Union[
    AristasSemicortas3,{Part[AristasSemicortas,i]}]]]]
Print["# AristasCortas= ",Length[AristasCortas],", Longitud m nima ",Min[Table[ArcCos[Part[
    AristasCortas,i,1].Part[AristasCortas,i,2]],{i,1,119}]],", Longitud promedio ",Mean[
    Table[ArcCos[Part[AristasCortas,i,1].Part[AristasCortas,i,2]],{i,1,119}]]]
Print["#AristasSemicortas1= ",Length[AristasSemicortas1],", Longitud promedio ",Mean[Table[
    ArcCos[Part[AristasSemicortas1,i,1].Part[AristasSemicortas1,i,2]],{i,1,9}]]]
Print["#AristasSemicortas2= ",Length[AristasSemicortas2],", Longitud promedio ",Mean[Table[
    ArcCos[Part[AristasSemicortas2,i,1].Part[AristasSemicortas2,i,2]],{i,1,8}]]]
Print["#AristasSemicortas3= ",Length[AristasSemicortas3],", Longitud promedio ",Mean[Table[
    ArcCos[Part[AristasSemicortas3,i,1].Part[AristasSemicortas3,i,2]],{i,1,2}]]]


Show[Graphics3D[{Red,Table[Triangle[Part[Y,i]],{i,1,Length[Y]}],White,Opacity[.3],Sphere[](*
    , Opacity[1],Black,Table[Inset[Text[Style[i,20]],(1.1)P[i]],{i,1,60}]*)},Boxed->False],
    ParametricPlot3D[Table[Arc[Part[AristasSemicortas1,i],t],{i,1,9}],{t,0,1},PlotStyle->
    Yellow],ParametricPlot3D[Table[Arc[Part[AristasSemicortas2,i],t],{i,1,8}],{t,0,1},
    PlotStyle->Green],ParametricPlot3D[Table[Arc[Part[AristasSemicortas3,i],t],{i,1,2}],{t
    ,0,1},PlotStyle->Blue]]

```

```
35
36 Durante la evaluaci\363n de In[551]:= Length[Y]= 48
37 Durante la evaluaci\363n de In[551]:= # AristasCortas= 119, Longitud m nima 0.47460441,
       Longitud promedio 0.47460442
38 Durante la evaluaci\363n de In[551]:= #AristasSemicortas1= 9, Longitud promedio 0.47547195
39 Durante la evaluaci\363n de In[551]:= #AristasSemicortas2= 8, Longitud promedio 0.48006082
40 #AristasSemicortas3= 2, Longitud promedio 0.49035004
41
42 Out[563]=
```



```
1  Arc[{A_,B_},t_]:=((1-t)A+t B)/Norm[((1-t)A+t B)]
2  Poligono1={{P[1],P[2]},{P[2],P[12]},{P[12],P[10]},{P[10],P[7]},{P[7],P[16]},{P[16],P[25]},{P
       [25],P[17]},{P[17],P[4]},{P[4],P[11]},{P[11],P[13]},{P[13],P[6]},{P[6],P[1]}};
3  Poligono2={{P[36],P[24]},{P[24],P[28]},{P[28],P[46]},{P[46],P[48]},{P[48],P[57]},{P[57],P
       [60]},{P[60],P[53]},{P[53],P[45]},{P[45],P[41]},{P[41],P[22]},{P[22],P[20]},{P[20],P
       [36]}};
4  Poligono3={{P[33],P[43]},{P[43],P[56]},{P[56],P[50]},{P[50],P[37]},{P[37],P[39]},{P[39],P
       [26]},{P[26],P[14]},{P[14],P[21]},{P[21],P[8]},{P[8],P[15]},{P[15],P[30]},{P[30],P
       [33]}};
5  Poligono4={{P[44],P[49]},{P[49],P[58]},{P[58],P[59]},{P[59],P[51]},{P[51],P[52]},{P[52],P
       [38]},{P[38],P[29]},{P[29],P[40]},{P[40],P[23]},{P[23],P[18]},{P[18],P[35]},{P[35],P
       [44]}};
6  PC[1]=RegionCentroid[ConvexHullRegion[Flatten[Poligono1,1]]];
7  PC[2]=RegionCentroid[ConvexHullRegion[Flatten[Poligono2,1]]];
8  PC[3]=RegionCentroid[ConvexHullRegion[Flatten[Poligono3,1]]];
9  PC[4]=RegionCentroid[ConvexHullRegion[Flatten[Poligono4,1]]];
10
11 {ArcCos[PC[1].PC[2]],ArcCos[PC[1].PC[3]],ArcCos[PC[1].PC[4]],ArcCos[PC[2].PC[3]],ArcCos[PC
       [2].PC[4]],ArcCos[PC[3].PC[4]]}
12
13 Show[Graphics3D[{Yellow,Sphere[],Black,Table[Inset[Text[Style[i,20]],(1.02)P[i]],{i
       ,1,60}]}],ParametricPlot3D[Table[Arc[Part[AristasCortas,i],t],{i,1,119}],{t,0,1},
       PlotStyle->Red],ParametricPlot3D[Table[Arc[Part[AristasSemicortas,i],t],{i,1,19}],{t
       ,0,1},PlotStyle->Yellow],ParametricPlot3D[Table[Arc[Part[AristasMedianas,i],t],{i
       ,1,24}],{t,0,1},PlotStyle->Blue],ParametricPlot3D[Table[Arc[Part[AristasLargas,i],t],{i
       ,1,12}],{t,0,1},PlotStyle->Green],ParametricPlot3D[Table[Arc[Part[Poligono1,i],t],{i
       ,1,12}],{t,0,1},PlotStyle->{LightGray,Thickness[.02]}],ParametricPlot3D[Table[Arc[Part[
       Poligono2,i],t],{i,1,12}],{t,0,1},PlotStyle->{LightGray,Thickness[.02]}],
       ParametricPlot3D[Table[Arc[Part[Poligono3,i],t],{i,1,12}],{t,0,1},PlotStyle->{LightGray,
       Thickness[.02]}],ParametricPlot3D[Table[Arc[Part[Poligono4,i],t],{i,1,12}],{t,0,1},
       PlotStyle->{LightGray,Thickness[.02]}],ParametricPlot3D[Arc[{PC[1],PC[2]},t],{t,0,1},
       PlotStyle->Black],ParametricPlot3D[Arc[{PC[1],PC[3]},t],{t,0,1},PlotStyle->Black],
```

```
    ParametricPlot3D[Arc[{PC[1],PC[4]},t],{t,0,1},PlotStyle->Black],ParametricPlot3D[Arc[{PC
    [2],PC[3]},t],{t,0,1},PlotStyle->Black],ParametricPlot3D[Arc[{PC[2],PC[4]},t],{t,0,1},
    PlotStyle->Black],ParametricPlot3D[Arc[{PC[3],PC[4]},t],{t,0,1},PlotStyle->Black]]
Out[573]= {1.73178,1.72475,1.72901,1.72901,1.72475,1.72672}
Out[574]=
```

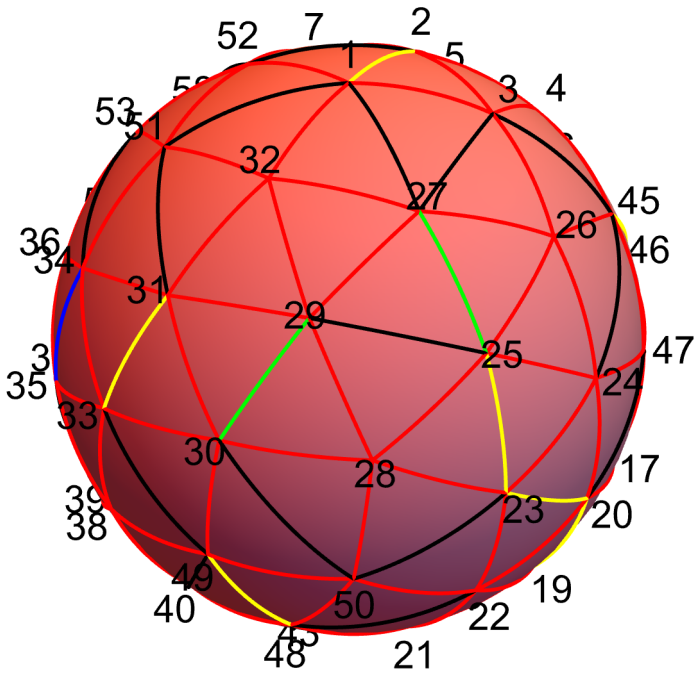


```

In[2]:= SetPrecision[\[Alpha],9]-SetPrecision[\[Alpha]0,9]
SetPrecision[ArcCos[\[Alpha]0],10]
SetPrecision[ArcCos[\[Alpha]],10]
2*10^(-9)+3*10^(-10)+6*10^(-11)+8*10^(-12.)
Out[2]= \[Alpha]-\[Alpha]0
Out[3]= ArcCos[\[Alpha]0]
Out[4]= ArcCos[\[Alpha]]
Out[5]= 2.368*10^-9
In[6]:= 1.07628177722176582531378105644925824436`10.1
    -1.07628177133472242588954279063351293644`10.
Out[6]= 6.*10^-9
In[138]:= \[Alpha]0=ArcCos[0.889473568664781383`8.688124857428225]
{\[Epsilon]c\[Alpha]=-2/10^9-3/10^10-6/10^11-8/10^12-9/10^13,(*\[Epsilon]c\[Beta
    ]1=-1/10^8-8/10^9-5/10^10-3/10^11-9/10^13,\[Epsilon]c\[Beta]2=0,\[Epsilon]c\[Beta
    ]3=-4/10^13,\[Epsilon]c\[Beta]4=-4/10^9-1/10^11-9/10^12-8/10^13,\[Epsilon]c\[Gamma
    ]1=0,\[Epsilon]c\[Gamma]2=0,\[Epsilon]c\[Gamma]3=0,\[Epsilon]c\[Gamma]4=0,\[Epsilon]c\[
    Delta]=0*)};
\[Epsilon]c\[Alpha]=2*10^(-9)+3*10^(-10)+6*10^(-11)+8*10^(-12);
\[Alpha]=ArcCos[-\[Epsilon]c\[Alpha]+Cos[\[Alpha]0]];
A=ArcCos[Cos[\[Alpha]]/(1+Cos[\[Alpha]])];
{\[Beta]1=ArcCos[0.8893185854087823867],\[Beta]2=ArcCos[0.8892732945383421843],\[Beta]3=
    ArcCos[0.8891003676635580072],\[Beta]4=ArcCos[0.8887814713774418999]};
{B1=ArcCos[(Cos[\[Beta]1]-Cos[\[Alpha]]^2)/Sin[\[Alpha]]^2],B2=ArcCos[(Cos[\[Beta]2]-Cos[\[
    Alpha]]^2)/Sin[\[Alpha]]^2],B3=ArcCos[(Cos[\[Beta]3]-Cos[\[Alpha]]^2)/Sin[\[Alpha]]^2],
    B4=ArcCos[(Cos[\[Beta]4]-Cos[\[Alpha]]^2)/Sin[\[Alpha]]^2]};
{\[Gamma]1=ArcCos[0.8880460352643052995],\[Gamma]2=ArcCos[0.8870695855434101028],\[Gamma]3=
```

```
    ArcCos[0.8864807165749388689],\[Gamma]4=ArcCos[0.8862670508008343837]};
{G1=ArcCos[(Cos[\[Gamma]1]-Cos[\[Alpha]]^2)/Sin[\[Alpha]]^2],G2=ArcCos[(Cos[\[Gamma]2]-Cos
    [\[Alpha]]^2)/Sin[\[Alpha]]^2],G3=ArcCos[(Cos[\[Gamma]3]-Cos[\[Alpha]]^2)/Sin[\[Alpha
    ]]^2],
G4=ArcCos[(Cos[\[Gamma]4]-Cos[\[Alpha]]^2)/Sin[\[Alpha]]^2]};
\[Delta]=ArcCos[0.8821680683605515049];
De=ArcCos[(Cos[\[Delta]]-Cos[\[Alpha]]^2)/Sin[\[Alpha]]^2];
Rz[\[Theta]_]:=(Cos[\[Theta]] -Sin[\[Theta]]  0
Sin[\[Theta]] Cos[\[Theta]] 0
0 0 1

);
Rx=(-1  0 0
0 1 0
0 0 1

);
Arc[A_,B_,t_]:=((1-t)A+t B)/Norm[((1-t)A+t B)]
Refl[x_,y_]:=ReflectionMatrix[Cross[x,y]]
Rot[x_,y_]:=Transpose[{Cross[y,x]/Norm[Cross[x,y]],(y-y.x x)/Norm[(y-y.x x)],x}]
T[1]={0,0,1};
T[2]={0,Sin[\[Alpha]],Cos[\[Alpha]]};
T[3]=Rz[-A].T[2];
Q[1]={0,0,1};
Q[2]={0,Sin[\[Beta]2],Cos[\[Beta]2]};
Q[3]={x,y,z}/.FindRoot[Q[1].{x,y,z}==Cos[\[Alpha]]&&Q[2].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z
    }]==1,{{x,0.40443374669294246},{y,0.2127677493950803},{z,0.8894735686647813}}];
Q[4]=Rot[Q[3],Q[2]].T[3];
Q[4]=Rot[Q[3],Q[2]].T[3];
Q[5]={x,y,z}/.FindRoot[Q[2].{x,y,z}==Cos[\[Alpha]]&&Q[4].{x,y,z}==Cos[\[Gamma]3]&&Norm[{x,y,
    z}]==1,{{x,-0.048902394720918146},{y,0.810876540954435},{z,0.5831704648902669}}];
Q[6]={x,y,z}/.FindRoot[Q[4].{x,y,z}==Cos[\[Alpha]]&&Q[5].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z
    }]==1,{{x,0.3081940854511294},{y,0.9037453729090233},{z,0.2970870354601019}}];
Q[7]=Rot[Q[5],Q[2]].T[3];
Q[8]={x,y,z}/.FindRoot[Q[7].{x,y,z}==Cos[\[Alpha]]&&Q[5].{x,y,z}==Cos[\[Gamma]2]&&Norm[{x,y,
    z}]==1,{{x,-0.4555950217704665},{y,0.8261716393795384},{z,0.3314718667140452}}];
Q[9]={x,y,z}/.FindRoot[Q[6].{x,y,z}==Cos[\[Alpha]]&&Q[8].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z
    }]==1,{{x,-0.09734895146906396},{y,0.9923204372889003},{z,0.07631075537982657}}];
Q[10]=Rot[Q[6],Q[9]].T[3];
Q[11]=Rot[Q[9],Q[8]].T[3];
Q[12]={x,y,z}/.FindRoot[Q[10].{x,y,z}==Cos[\[Alpha]]&&Q[11].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,
    y,z}]==1,{{x,-0.11041521747336673},{y,0.9090597828639854},{z,-0.4017695744199567}}];
Q[13]={x,y,z}/.FindRoot[Q[12].{x,y,z}==Cos[\[Alpha]]&&Q[10].{x,y,z}==Cos[\[Beta]1]&&Norm[{x,
    y,z}]==1,{{x,0.2834468762504213},{y,0.7472186148824155},{z,-0.6011008317388046}}];
Q[14]={x,y,z}/.FindRoot[Q[10].{x,y,z}==Cos[\[Alpha]]&&Q[13].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,
    y,z}]==1,{{x,0.6583718228692461},{y,0.6785039712462315},{z,-0.325851045502265}}];
Q[15]={x,y,z}/.FindRoot[Q[13].{x,y,z}==Cos[\[Alpha]]&&Q[14].{x,y,z}==Cos[\[Gamma]4]&&Norm[{x
    ,y,z}]==1,{{x,0.5834596726375917},{y,0.4008475640272482},{z,-0.7063257328025472}}];
Q[16]=Rot[Q[15],Q[13]].T[3];
Q[17]={x,y,z}/.FindRoot[Q[14].{x,y,z}==Cos[\[Alpha]]&&Q[15].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,
    y,z}]==1,{{x,0.8877301228068757},{y,0.2707227858496931},{z,-0.3723498385699538}}];
Q[18]=Rot[Q[15],Q[16]].T[3];
Q[19]={x,y,z}/.FindRoot[Q[17].{x,y,z}==Cos[\[Alpha]]&&Q[18].{x,y,z}==Cos[\[Gamma]1]&&Norm[{x
    ,y,z}]==1,{{x,0.7711273309577059},{y,-0.10642600147777967},{z,-0.6277229848105742}}];
Q[20]={x,y,z}/.FindRoot[Q[17].{x,y,z}==Cos[\[Alpha]]&&Q[19].{x,y,z}==Cos[\[Beta]4]&&Norm[{x,
    y,z}]==1,{{x,0.9662341298966517},{y,-0.1602988130216733},{z,-0.2017322402758248}}];
Q[21]={x,y,z}/.FindRoot[Q[18].{x,y,z}==Cos[\[Alpha]]&&Q[19].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,
    y,z}]==1,{{x,0.464748763786155},{y,-0.4322214941612015},{z,-0.7727827421366883}}];
Q[22]=Rot[Q[19],Q[21]].T[3];
Q[23]={x,y,z}/.FindRoot[Q[22].{x,y,z}==Cos[\[Alpha]]&&Q[20].{x,y,z}==Cos[\[Beta]3]&&Norm[{x,
    y,z}]==1,{{x,0.8418337464568276},{y,-0.5372551413485798},{z,0.05169967525023173}}];
Q[24]={x,y,z}/.FindRoot[Q[20].{x,y,z}==Cos[\[Alpha]]&&Q[23].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,
    y,z}]==1,{{x,0.9540374286644094},{y,0.13494235048096523},{z,0.2675876431266477}}];
Q[25]={x,y,z}/.FindRoot[Q[24].{x,y,z}==Cos[\[Alpha]]&&Q[23].{x,y,z}==Cos[\[Beta]4]&&Norm[{x,
    y,z}]==1,{{x,0.7245097631148443},{y,-0.470591925476885},{z,0.5036157690414689}}];
Q[26]=Rot[Q[24],Q[25]].T[3];
Q[27]={x,y,z}/.FindRoot[Q[26].{x,y,z}==Cos[\[Alpha]]&&Q[25].{x,y,z}==Cos[\[Gamma]1]&&Norm[{x
    ,y,z}]==1,{{x,0.42235469906842704},{y,-0.33583457084856505},{z,0.84192140321872}}];
```

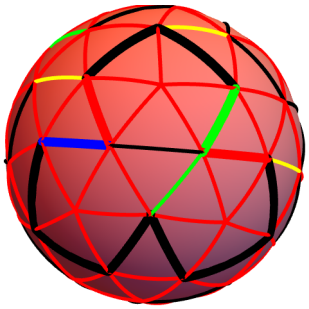
```
Q[28]={x,y,z}/.FindRoot[Q[23].{x,y,z}==Cos[\[Alpha]]&&Q[25].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,0.5212218030958334},{y,-0.8142975068370093},{z,0.2554357107699687}}];
Q[29]={x,y,z}/.FindRoot[Q[27].{x,y,z}==Cos[\[Alpha]]&&Q[28].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,0.2600312007223925},{y,-0.726266982962316},{z,0.6363332806868474}}];
Q[30]={x,y,z}/.FindRoot[Q[28].{x,y,z}==Cos[\[Alpha]]&&Q[29].{x,y,z}==Cos[\[Gamma]4]&&Norm[{x,y,z}]==1,{{x,0.07474790554042843},{y,-0.9613421183385972},{z,0.26501713553198375}}];Q[31]={x,y,z}/.FindRoot[Q[29].{x,y,z}==Cos[\[Alpha]]&&Q[30].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.2054900871274366},{y,-0.777294403977514},{z,0.594632015315018}}];
Q[32]=Rot[Q[29],Q[31]].T[3];Q[33]={x,y,z}/.FindRoot[Q[30].{x,y,z}==Cos[\[Alpha]]&&Q[31].{x,y,z}==Cos[\[Beta]1]&&Norm[{x,y,z}]==1,{{x,-0.3840889961505356},{y,-0.9058248379226465},{z,0.17876522603259656}}];
Q[34]={x,y,z}/.FindRoot[Q[31].{x,y,z}==Cos[\[Alpha]]&&Q[33].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.6298845725519405},{y,-0.6234515635140365},{z,0.463199280237958}}];
Q[35]={x,y,z}/.FindRoot[Q[33].{x,y,z}==Cos[\[Alpha]]&&Q[34].{x,y,z}==Cos[\[Delta]]&&Norm[{x,y,z}]==1,{{x,-0.744577923672671},{y,-0.6675044103325863},{z,-0.006448082338236139 6}}];
Q[36]={x,y,z}/.FindRoot[Q[34].{x,y,z}==Cos[\[Alpha]]&&Q[35].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.9149959071202968},{y,-0.3143331664012362},{z,0.25293704839994824}}];
Q[37]=Rot[Q[36],Q[35]].T[3];
Q[38]=Rot[Q[35],Q[33]].T[3];
Q[39]={x,y,z}/.FindRoot[Q[38].{x,y,z}==Cos[\[Alpha]]&&Q[37].{x,y,z}==Cos[\[Gamma]2]&&Norm[{x,y,z}]==1,{{x,-0.6635200981536724},{y,-0.5340800651215074},{z,-0.5239270592228916}}];
Q[40]={x,y,z}/.FindRoot[Q[38].{x,y,z}==Cos[\[Alpha]]&&Q[39].{x,y,z}==Cos[\[Gamma]3]&&Norm[{x,y,z}]==1,{{x,-0.24204679638935747},{y,-0.6860388999293006},{z,-0.686122420666635 8}}];
Q[41]={x,y,z}/.FindRoot[Q[40].{x,y,z}==Cos[\[Alpha]]&&Q[39].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.4330561956675092},{y,-0.2903028108925441},{z,-0.853338508085667}}];
Q[42]=Rot[Q[39],Q[41]].T[3];
Q[43]=Rot[Q[41],Q[40]].T[3];
Q[44]={x,y,z}/.FindRoot[Q[43].{x,y,z}==Cos[\[Alpha]]&&Q[41].{x,y,z}==Cos[\[Beta]2]&&Norm[{x,y,z}]==1,{{x,-0.14359695911048806},{y,0.055399817080754414},{z,-0.9880843959913749}}];
Q[45]={x,y,z}/.FindRoot[Q[26].{x,y,z}==Cos[\[Alpha]]&&Q[4].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,0.7579151060535679},{y,0.4080571968885414},{z,0.5089734925153518}}];
Q[46]={x,y,z}/.FindRoot[Q[6].{x,y,z}==Cos[\[Alpha]]&&Q[14].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,0.7015949106533736},{y,0.6983040187998999},{z,0.14190165141108838}}];
Q[47]={x,y,z}/.FindRoot[Q[17].{x,y,z}==Cos[\[Alpha]]&&Q[24].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,0.9489427036305138},{y,0.3014911931657835},{z,0.09279442693331792}}];
Q[48]={x,y,z}/.FindRoot[Q[21].{x,y,z}==Cos[\[Alpha]]&&Q[40].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,0.2081461962717046},{y,-0.7807643815405613},{z,-0.5891367765597442}}];
Q[49]={x,y,z}/.FindRoot[Q[30].{x,y,z}==Cos[\[Alpha]]&&Q[38].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,0.02593583349519103},{y,-0.9789859321890553},{z,-0.2022717902151371}}];
Q[50]={x,y,z}/.FindRoot[Q[22].{x,y,z}==Cos[\[Alpha]]&&Q[28].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,0.47860298591121175},{y,-0.8522394055135843},{z,-0.2112514557742139}}];Q[51]={x,y,z}/.FindRoot[Q[32].{x,y,z}==Cos[\[Alpha]]&&Q[34].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.4786754662045244},{y,-0.34241957281472607},{z,0.8084668417487894}}];Q[52]={x,y,z}/.FindRoot[Q[1].{x,y,z}==Cos[\[Alpha]]&&Q[7].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.4411692849625412},{y,0.11919074063189028},{z,0.8894735686647813}}];
Q[53]={x,y,z}/.FindRoot[Q[51].{x,y,z}==Cos[\[Alpha]]&&Q[52].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.7294466145870048},{y,-0.174047454315023},{z,0.6615250652035971}}];
Q[54]={x,y,z}/.FindRoot[Q[42].{x,y,z}==Cos[\[Alpha]]&&Q[44].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.5295804439302658},{y,0.25532524296545095},{z,-0.8089212407343844}}];
Q[55]={x,y,z}/.FindRoot[Q[16].{x,y,z}==Cos[\[Alpha]]&&Q[12].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.2371607412198595},{y,0.6185897230073244},{z,-0.7490671114217794}}];
Q[56]={x,y,z}/.FindRoot[Q[54].{x,y,z}==Cos[\[Alpha]]&&Q[55].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.6336837840139935},{y,0.592126667243545},{z,0.49782614617631193}}];
Q[58]={x,y,z}/.FindRoot[Q[53].{x,y,z}==Cos[\[Alpha]]&&Q[7].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.7770574165230251},{y,0.28303198713070393},{z,0.5622051811282133}}];
Q[60]={x,y,z}/.FindRoot[Q[42].{x,y,z}==Cos[\[Alpha]]&&Q[56].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.91156030907628},{y,0.2563762021926282},{z,-0.3214483564525433}}];
Q[57]={x,y,z}/.FindRoot[Q[58].{x,y,z}==Cos[\[Alpha]]&&Q[60].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.9187422383471895},{y,0.14134119579867604},{z,0.3686941358999306}}];
Q[59]={x,y,z}/.FindRoot[Q[58].{x,y,z}==Cos[\[Alpha]]&&Q[60].{x,y,z}==Cos[\[Alpha]]&&Norm[{x,y,z}]==1,{{x,-0.7975241230868134},{y,0.5935818022969545},{z,0.10777716398434424}}];
Malas={};
For[i=1,i<60,i++,For[j=i+1,j<=60,j++,If[Q[i].Q[j]>Cos[\[Alpha]],Malas=Union[Malas,{{i,j,Q[i].Q[j]-Cos[\[Alpha]]}}]]]]
Print["Malas = ",Malas]
A\[Alpha]={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Cos[\[Alpha]]-10^(-8)<=Q[i].Q[j]<=Cos[\[Alpha]],A\[Alpha]=Union[A\[Alpha] ,{{i,j}}]]]]
```

```
A\[Beta]={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Cos[\[Beta]4]<=Q[i].Q[j]<=Cos[\[Beta]1],A\[Beta]=
    Union[A\[Beta] ,{{i,j}}]]]]
A\[Gamma]={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Cos[\[Gamma]4]<=Q[i].Q[j]<=Cos[\[Gamma]1],A\[Gamma
    ]=Union[A\[Gamma] ,{{i,j}}]]]]
A\[Delta]={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Cos[\[Delta]]<=Q[i].Q[j]<=Cos[\[Delta]]+10^(-8),A\[
    Delta]=Union[A\[Delta] ,{{i,j}}]]]]
AL={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[0.79<Q[i].Q[j]<Cos[\[Delta]],AL=Union[AL ,{{i,j
    }}]]]]
Print["A\[Alpha] = ",Length[A\[Alpha]],"   A\[Beta] = ",Length[A\[Beta]],"   A\[Gamma] = ",
    Length[A\[Gamma]],"   A\[Delta] = ",Length[A\[Delta]],"   AL = ",Length[AL]]
Poligono1={{Q[35],Q[33]},{Q[33],Q[49]},{Q[49],Q[40]},{Q[40],Q[39]},{Q[39],Q[41]},{Q[41],Q
    [54]},{Q[54],Q[60]},{Q[60],Q[37]},{Q[37],Q[57]},{Q[57],Q[53]},{Q[53],Q[34]},{Q[34],Q
    [35]}};
Poligono2={{Q[8],Q[59]},{Q[59],Q[56]},{Q[56],Q[12]},{Q[12],Q[9]},{Q[9],Q[10]},{Q[10],Q
    [46]},{Q[46],Q[4]},{Q[4],Q[5]},{Q[5],Q[2]},{Q[2],Q[52]},{Q[52],Q[58]},{Q[58],Q[8]}};
Poligono3={{Q[27],Q[3]},{Q[3],Q[45]},{Q[45],Q[24]},{Q[24],Q[25]},{Q[25],Q[23]},{Q[23],Q
    [50]},{Q[50],Q[30]},{Q[30],Q[29]},{Q[29],Q[31]},{Q[31],Q[51]},{Q[51],Q[1]},{Q[1],Q
    [27]}};
Poligono4={{Q[19],Q[20]},{Q[20],Q[47]},{Q[47],Q[14]},{Q[14],Q[15]},{Q[15],Q[13]},{Q[13],Q
    [55]},{Q[55],Q[44]},{Q[44],Q[18]},{Q[18],Q[43]},{Q[43],Q[48]},{Q[48],Q[22]},{Q[22],Q
    [19]}};
PC[1]=RegionCentroid[ConvexHullRegion[Flatten[Poligono1,1]]];
PC[2]=RegionCentroid[ConvexHullRegion[Flatten[Poligono2,1]]];
PC[3]=RegionCentroid[ConvexHullRegion[Flatten[Poligono3,1]]];
PC[4]=RegionCentroid[ConvexHullRegion[Flatten[Poligono4,1]]];
{ArcCos[PC[1].PC[2]],ArcCos[PC[1].PC[3]],ArcCos[PC[1].PC[4]],ArcCos[PC[2].PC[3]],ArcCos[PC
    [2].PC[4]],ArcCos[PC[3].PC[4]]};

Show[Graphics3D[{Opacity[1],Pink,Sphere[],Opacity[1],Black,Table[Inset[Text[Style[i
    ,20]],(1.1)Q[i]],{i,1,60}]},Boxed->False],Table[ParametricPlot3D[Arc[Q[Part[A\[Alpha],i
    ,1]],Q[Part[A\[Alpha],i,2]],t],{t,0,1},PlotStyle->Red],{i,1,Length[A\[Alpha]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Beta],i,1]],Q[Part[A\[Beta],i,2]],t],{t,0,1},PlotStyle
    ->Yellow],{i,1,Length[A\[Beta]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Gamma],i,1]],Q[Part[A\[Gamma],i,2]],t],{t,0,1},
    PlotStyle->Green],{i,1,Length[A\[Gamma]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Delta],i,1]],Q[Part[A\[Delta],i,2]],t],{t,0,1},
    PlotStyle->Blue],{i,1,Length[A\[Delta]]}],
Table[ParametricPlot3D[Arc[Q[Part[AL,i,1]],Q[Part[AL,i,2]],t],{t,0,1},PlotStyle->Black],{i
    ,1,Length[AL]}],ParametricPlot3D[Table[Arc[Part[Poligono1,i],t],{i,1,12}],{t,0,1},
    PlotStyle->{LightGray,Thickness[.01]}],ParametricPlot3D[Table[Arc[Part[Poligono2,i],t],{
    i,1,12}],{t,0,1},PlotStyle->{LightGray,Thickness[.01]}],ParametricPlot3D[Table[Arc[Part[
    Poligono3,i],t],{i,1,12}],{t,0,1},PlotStyle->{LightGray,Thickness[.01]}],
    ParametricPlot3D[Table[Arc[Part[Poligono4,i],t],{i,1,12}],{t,0,1},PlotStyle->{LightGray,
    Thickness[.01]}],ParametricPlot3D[Arc[{PC[1],PC[2]},t],{t,0,1},PlotStyle->Black],
    ParametricPlot3D[Arc[{PC[1],PC[3]},t],{t,0,1},PlotStyle->Black],ParametricPlot3D[Arc[{PC
    [1],PC[4]},t],{t,0,1},PlotStyle->Black],ParametricPlot3D[Arc[{PC[2],PC[3]},t],{t,0,1},
    PlotStyle->Black],ParametricPlot3D[Arc[{PC[2],PC[4]},t],{t,0,1},PlotStyle->Black],
    ParametricPlot3D[Arc[{PC[3],PC[4]},t],{t,0,1},PlotStyle->Black]]
Out[138]= 0.47460441
Durante la evaluaci\363n de In[138]:= Malas = {}
Durante la evaluaci\363n de In[138]:= A\[Alpha] = 119   A\[Beta] = 9   A\[Gamma] = 8   A\[
    Delta] = 2   AL = 36
Out[237]=
```

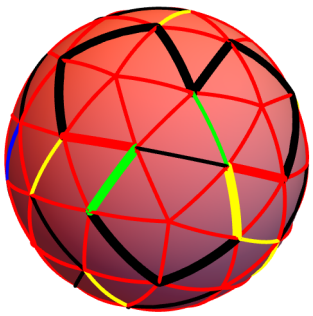



```
In[107]:= A\[Alpha]={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Cos[\[Alpha]]-10^(-8)<=Q[i].Q[j]<=Cos[\[Alpha]],A\[
    Alpha]=Union[A\[Alpha],{{i,j}}]]]]
A\[Beta]1={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[ Q[i].Q[j]==Cos[\[Beta]1],A\[Beta]1=Union[A\[Beta]1
    ,{{i,j}}]]]]
A\[Beta]2={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[ Q[i].Q[j]==Cos[\[Beta]2],A\[Beta]2=Union[A\[Beta]2
    ,{{i,j}}]]]]
A\[Beta]3={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Cos[\[Beta]3]-3*10^(-4)<Q[i].Q[j]<=Cos[\[Beta]3],A
    \[Beta]3=Union[A\[Beta]3 ,{{i,j}}]]]]
A\[Beta]4={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[ Q[i].Q[j]==Cos[\[Beta]4],A\[Beta]4=Union[A\[Beta]4
    ,{{i,j}}]]]]
A\[Gamma]1={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Q[i].Q[j]==Cos[\[Gamma]1],A\[Gamma]1=Union[A\[Gamma
    ]1 ,{{i,j}}]]]]
A\[Gamma]2={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Q[i].Q[j]==Cos[\[Gamma]2],A\[Gamma]2=Union[A\[Gamma
    ]2 ,{{i,j}}]]]]
A\[Gamma]3={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Q[i].Q[j]==Cos[\[Gamma]3],A\[Gamma]3=Union[A\[Gamma
    ]3 ,{{i,j}}]]]]
A\[Gamma]4={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Q[i].Q[j]==Cos[\[Gamma]4],A\[Gamma]4=Union[A\[Gamma
    ]4 ,{{i,j}}]]]]
A\[Delta]={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[Cos[\[Delta]]<=Q[i].Q[j]<=Cos[\[Delta]]+10^(-8),A\[
    Delta]=Union[A\[Delta] ,{{i,j}}]]]]
AL={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[0.79<Q[i].Q[j]<Cos[\[Delta]],AL=Union[AL ,{{i,j
    }}]]]]
AT={};
For[i=1,i<=59,i++,For[j=i+1,j<=60,j++,If[0.79<Q[i].Q[j]<=Cos[\[Alpha]],AT=Union[AT ,{{i,j
    }}]]]]

Print["A\[Alpha] = ",Length[A\[Alpha]],"   A\[Beta]1 = ",Length[A\[Beta]1],"   A\[Beta]2 = "
```

```
    ,Length[A\[Beta]2],"   A\[Beta]3 = ",Length[A\[Beta]3],"   A\[Beta]4 = ",Length[A\[Beta
    ]4],"   A\[Gamma]1 = ",Length[A\[Gamma]1],"   A\[Gamma]2 = ",Length[A\[Gamma]2],"   A\[
    Gamma]3 = ",Length[A\[Gamma]3],"   A\[Gamma]4 = ",Length[A\[Gamma]4],"   A\[Delta] = ",
    Length[A\[Delta]],"   AL = ",Length[AL],"   AT = ",Length[AT]]
Durante la evaluaci\363n de In[107]:= A\[Alpha] = 119   A\[Beta]1 = 2   A\[Beta]2 = 2   A\[
    Beta]3 = 3   A\[Beta]4 = 2   A\[Gamma]1 = 2   A\[Gamma]2 = 2   A\[Gamma]3 = 2   A\[Gamma
    ]4 = 2   A\[Delta] = 2   AL = 36   AT = 174
In[132]:= Show[Graphics3D[{Opacity[1],Pink,Sphere[]},Boxed->False],Table[ParametricPlot3D[
    Arc[Q[Part[A\[Alpha],i,1]],Q[Part[A\[Alpha],i,2]],t],{t,0,1},PlotStyle->Red],{i,1,Length
    [A\[Alpha]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Beta],i,1]],Q[Part[A\[Beta],i,2]],t],{t,0,1},PlotStyle
    ->Yellow],{i,1,Length[A\[Beta]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Gamma],i,1]],Q[Part[A\[Gamma],i,2]],t],{t,0,1},
    PlotStyle->Green],{i,1,Length[A\[Gamma]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Delta],i,1]],Q[Part[A\[Delta],i,2]],t],{t,0,1},
    PlotStyle->Blue],{i,1,Length[A\[Delta]]}],
Table[ParametricPlot3D[Arc[Q[Part[AL,i,1]],Q[Part[AL,i,2]],t],{t,0,1},PlotStyle->Black],{i
    ,1,Length[AL]}],ParametricPlot3D[Arc[Q[37],Q[57],t],{t,0,1},PlotStyle->{Thickness[.03],
    Black}],ParametricPlot3D[Arc[Q[57],Q[53],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[53],Q[34],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[34],Q[35],t],{t,0,1},PlotStyle->{Thickness[.03],Blue}],
    ParametricPlot3D[Arc[Q[35],Q[33],t],{t,0,1},PlotStyle->{Thickness[.03],Red}],
    ParametricPlot3D[Arc[Q[33],Q[49],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[49],Q[40],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[40],Q[39],t],{t,0,1},PlotStyle->{Thickness[.03],Green}],
    ParametricPlot3D[Arc[Q[39],Q[41],t],{t,0,1},PlotStyle->{Thickness[.03],Red}],
    ParametricPlot3D[Arc[Q[41],Q[54],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[54],Q[60],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[60],Q[37],t],{t,0,1},PlotStyle->{Thickness[.03],Black}]]
```



```

Show[Graphics3D[{Opacity[1],Pink,Sphere[]},Boxed->False],Table[ParametricPlot3D[Arc[Q[Part[A
    \[Alpha],i,1]],Q[Part[A\[Alpha],i,2]],t],{t,0,1},PlotStyle->Red],{i,1,Length[A\[Alpha
    ]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Beta],i,1]],Q[Part[A\[Beta],i,2]],t],{t,0,1},PlotStyle
    ->Yellow],{i,1,Length[A\[Beta]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Gamma],i,1]],Q[Part[A\[Gamma],i,2]],t],{t,0,1},
    PlotStyle->Green],{i,1,Length[A\[Gamma]]}],
Table[ParametricPlot3D[Arc[Q[Part[A\[Delta],i,1]],Q[Part[A\[Delta],i,2]],t],{t,0,1},
    PlotStyle->Blue],{i,1,Length[A\[Delta]]}],
Table[ParametricPlot3D[Arc[Q[Part[AL,i,1]],Q[Part[AL,i,2]],t],{t,0,1},PlotStyle->Black],{i
    ,1,Length[AL]}],ParametricPlot3D[Arc[Q[23],Q[50],t],{t,0,1},PlotStyle->{Thickness[.03],
    Black}],ParametricPlot3D[Arc[Q[50],Q[30],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[30],Q[29],t],{t,0,1},PlotStyle->{Thickness[.03],Green}],
    ParametricPlot3D[Arc[Q[29],Q[31],t],{t,0,1},PlotStyle->{Thickness[.03],Red}],
    ParametricPlot3D[Arc[Q[31],Q[51],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[51],Q[1],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[1],Q[27],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[27],Q[3],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[3],Q[45],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[45],Q[24],t],{t,0,1},PlotStyle->{Thickness[.03],Black}],
    ParametricPlot3D[Arc[Q[24],Q[25],t],{t,0,1},PlotStyle->{Thickness[.03],Red}],
    ParametricPlot3D[Arc[Q[25],Q[23],t],{t,0,1},PlotStyle->{Thickness[.03],Yellow}]]
Out[133]=
```



```
In[134]:= Grid[Table[{"Q",i,SetPrecision[Part[Q[i],1],9],SetPrecision[Part[Q[i],2],9],
    SetPrecision[Part[Q[i],3],9]},{i,1,60}]];
In[135]:= (* Tri ngulos m nimos *)
Y={};
For[i=1,i<=60,i++,
For[j=i+1,j<=60,j++,For[k=j+1,k<=60,k++,If[MemberQ[A\[Alpha],{i,j}]&&MemberQ[A\[Alpha],{i,k
    }]&&MemberQ[A\[Alpha],{j,k}],Y=Union[Y,{{i,j,k}}]]]]]
Print["Length[Y]= ",Length[Y]]
Durante la evaluaci\363n de In[135]:= Length[Y]= 18
```