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N̈orlund-Riesz Mean of Complex Uncertain Random Variables

Media N̈orlund-Riesz de Variables Aleatorias Inciertas Complejas

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Abstract. In this paper, we procured the notion of N̈orlund-Riesz Mean of Complex Uncertain Random Variables. Furthermore, some results on oscillating sequences of complex uncertain random variables are proved.

Keywords. N̈orlund-Riesz mean of complex uncertain random variables; convergence of complex uncertain random variables; uncertain theory.

Resumen. En este artículo, desarrollamos la noción de la media ”Norlund-Riesz de variables aleatorias inciertas complejas. Además, se demuestran algunos resultados sobre secuencias oscilatorias de variables aleatorias inciertas complejas.

Palabras Claves. variables aleatorias inciertas complejas, media N̈orlund-Riesz de variables aleatorias inciertas complejas, convergencia de variables aleatorias inciertas complejas, teoría de la incertidumbre complex uncertain random variables.

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1. Introduction

To possess knowledge about any form of information is often reflected through the way individuals express themselves linguistically—such as estimating a person's weight as “about 80 pounds,” referring to the “average age” of a population, or describing the “size” of an object. While these expressions may seem subjective or vague at first glance, studies have shown that such approximations do not behave purely randomly. Instead, they represent a deeper level of structured uncertainty that arises from human judgment and reasoning.

In response to this, Liu introduced the concept of uncertainty theory in 2009 [8] to mathematically handle this type of uncertainty, which is distinct from classical probability. Since its inception, uncertainty theory has found growing relevance across various domains of mathematics and applied sciences, offering powerful tools to model imprecise data, expert estimations, and systems involving human cognition and decision-making.

Today, uncertainty theory is being actively explored in areas such as decision analysis, artificial intelligence, engineering, economics, and information systems, where conventional

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probability models fall short. In particular, the study of convergence sequences under uncertainty has become vital, as convergence plays a crucial role in the behavior and analysis of algorithms, data streams, and prediction models based on uncertain information. Several researchers have investigated these concepts from multiple mathematical perspectives [10, 11, 6, 5].

A notable advancement in this field was made by Gao et al. in 2018 [4], who introduced the idea of complex uncertain random variables a generalization that allows for modeling uncertainty in complex-valued domains, with potential applications in quantum computing, signal processing, and complex systems. Later, Cheng et al. (2020) [3] extended this work by analyzing dispersion measures and pseudo-variance of these variables. Ahmadzade (2016) [2] further contributed by defining convergence for complex uncertain random variables and establishing their foundational properties.

In this paper, we introduce the concept of the Ňorlund-Riesz mean of complex uncertain random variables, a novel approach that offers more nuanced insights into their convergence behavior. Additionally, we present several results concerning oscillating sequences within this framework. These contributions not only extend the theoretical understanding of uncertainty in the complex domain but also open up new avenues for future research and application in fields requiring robust modeling of vague or imprecise phenomena.

2. Preliminaries

In this section, some well-known definitions are presented which will be useful for the development of this paper.

Understanding formal definitions is fundamental in the study of mathematics and logic. These definitions serve as the building blocks for constructing rigorous arguments and proofs. They allow us to communicate complex ideas with clarity and precision.

However, definitions are not just formalities. They often capture subtle nuances and lay the groundwork for deeper insights. To help the reader appreciate their importance and application, we provide simple illustrative examples alongside each definition.

Therefore, we present some basics notion on uncertain variables which were defined by [7, 8].

Definition 1. Let $\mathfrak{L}$ be a σ-algebra of non-empty set Ξ. A set function $\mathfrak{M}$ is said to be an uncertain measure if the following properties are satisfied:

(i) $\mathfrak{M}\{\Xi\} = 1$,

(ii) $\mathfrak{M}\{\Theta\} + \mathfrak{M}\{\Theta^c\} = 1$ for any $\Theta \in \mathfrak{L}$,

(iii) For every countable sequence of $\{\Omega_k\} \in \mathfrak{L}$, we obtain

$$\mathfrak{M}\left\{\bigcup_{k=1}^{\infty} \Theta_k\right\} \leq \sum_{k=1}^{\infty} \mathfrak{M}\{\Theta_k\}.$$

The triplet $(\Xi, \mathfrak{L}, \mathfrak{M})$ is said to be an uncertain space and each element $\Theta \in \mathfrak{L}$ is said to be an event.

Definition 2. Let $(\Xi_i, \mathfrak{L}_i, \mathfrak{M}_i)$ be uncertainty spaces for $i = 1, 2, ...$ The product uncertainty measure $\mathfrak{M}$ is an uncertain measure which satisfies:

$$\mathfrak{M}\left\{\prod_{i=1}^{\infty} \Theta_i\right\} = \bigwedge_{i=1}^{\infty} \mathfrak{M}_i\{\Theta_i\}.$$

Where Θ_i are arbitrarily events from $\mathfrak{L}_i$ for $i = 1, 2, ...$, respectively.

Definition 3. The change space refers to the product of $(\Xi, \mathfrak{L}, \mathfrak{M}) \times (\Omega, \mathfrak{A}, \mathfrak{P})$ where $(\Xi, \mathfrak{L}, \mathfrak{M})$ is an uncertain space and $(\Omega, \mathfrak{A}, \mathfrak{P})$ is a probability space. Let $\Delta \in \mathfrak{L} \times \mathfrak{M}$ be an uncertain random event. Then, the change measure of Δ is given by

$$Ch\{\Delta\} = \int_0^1 \mathfrak{P}\{\beta \in \Omega | \mathfrak{M}\{\alpha \in \Xi | (\alpha, \beta) \in \Delta\} \geq x\} dx.$$

Next, we present some notions on complex uncertain random variables.

Definition 4. ([4]) A complex uncertain random variable is a function ψ from a change space $(\Xi \times \Omega, \mathfrak{L} \times \mathfrak{A}, \mathfrak{M} \times \mathfrak{P})$ to the set of complex numbers such that $\{\psi \in \mathfrak{B}\} = \{(\alpha,\beta) \in \Xi \times \Omega | \psi(\alpha,\beta) \in \mathfrak{B}\}$ is an event in $\Xi \times \Omega$ for any Borel set $\mathfrak{B}$ of complex numbers

Definition 5. ([2]) The complex uncertain random sequence $\{\psi_n : n \geq 1\}$ is called almost surely convergent (a.s.) to a complex uncertain random variable ψ of there exists an event Δ with $Ch\{\Delta\} = 1$ such that

$$\lim_{n\to\infty} \|\psi_n(\alpha,\beta) - \psi(\alpha,\beta)\| = 0,$$

for every $(\alpha,\beta) \in \Delta$.

Definition 6. ([1]) The complex uncertain random sequence $\{\psi_n : n \geq 1\}$ is called convergent in measure to ψ if

$$\lim_{n\to\infty} Ch\{(\alpha,\beta) \in \Delta | \|\psi_n(\alpha,\beta) - \psi(\alpha,\beta)\| \geq \varepsilon\} = 0,$$

for any $\varepsilon > 0$.

Definition 7. ([1]) The complex uncertain random sequence $\{\psi_n : n \geq 1\}$ is called convergent in mean to ψ if

$$\lim_{n\to\infty} E\|\psi_n - \psi\| = 0.$$

We shall recall that $\psi_n = \zeta_n + i\kappa_n$ and $\psi = \zeta + i\kappa$, where $\|\psi_n - \psi\|$ is found as follows

$$\|\psi_n - \psi\| = \sqrt{|\zeta_n - \zeta|^2 + |\kappa_n - \kappa|^2}$$

where $\zeta, zeta_1, ...$ are uncertain variables and $\kappa, \kappa_1, ...$ are random variables.

Next, let $\mathfrak{O} = (o_{ni})$ be an infinite matrix mapping of the sequence space $\mathfrak{E}$ into a sequence space $\mathfrak{F}$. Thus, if $\varpi = \{\varpi_n\} \in \mathfrak{E}$, the $\mathfrak{O}$-transform of $\{\varpi_n\}$ is the sequence $(\mathfrak{O}_n(\varpi))$, where $\mathfrak{O}_n\varpi = \sum_{i=1}^{\infty} o_{ni}\varpi_i$ for each $n \in \mathbb{N}$ provided the summation exists for each $n \in \mathbb{N}$

Lemma 8. ([9]) The matrix $\mathfrak{O} = (o_{mn})$ is regular or limit preserving if and only if the following conditions holds:

(i) There exists a constant U such that $\sum_{n=1}^{\infty} |o_{mn}| < U$ for every m,

(ii) for every n, $\lim_{n\to\infty} o_{mn} = 0$,

(iii) $\lim_{m\to\infty} \sum_{n=1}^{\infty} o_{mn} = 1$.

Now, we shall remain the notion of N̈orlund and Riesz mean. Throughout this paper, $\{v_n\}$ will be represented a non-negative sequence of real numbers in particular $v_1 > 0$ and $V_n = v_1 + ... + v_n$ for $n = 1, 2, ...$ The transform defined by $y_n = \frac{v_m e_1 + ... + v_1 e_m}{V_m}$ is said to be the N̈orlund mean $(\mathfrak{N}, v_n)$. Also, the matrix of the $(\mathfrak{N}, v_n)$ mean is denoted by

$$o_{mn} = \begin{cases} \frac{v_{m-n+1}}{V_m}, & n \leq m \\ 0, & n > m. \end{cases}$$

Finally, the transformation defined by $y_n = \frac{v_m e_1 + ... + v_1 e_m}{V_m}$ is said to be Riesz mean or $(\mathfrak{R}, v_n)$ mean. the matrix of the $(\mathfrak{R}, v_n)$ is defined by

$$o_{mn} = \begin{cases} \frac{v_n}{V_m}, & n \leq m \\ 0, & n > m. \end{cases}$$

3. Main Results

In this section, we present the principal results obtained, the definitions presented are fundamental to the development of the work as they establish the theoretical framework necessary to analyze the convergence properties of complex uncertain random sequences under generalized summability methods. Specifically:

- **Definition 9** introduces the sequence $\psi = \{\psi_n\}$ of complex uncertain random variables and the auxiliary sequence $v = \{v_n\}$, both defined in the product measurable space $(\Xi \times \Omega, \mathfrak{L} \times \mathfrak{A}, \mathfrak{M} \times \mathfrak{P})$. This sets the probabilistic and uncertain environment in which the study is conducted.
- **Definition 10** defines the Riesz mean, a summability method that generalizes the notion of convergence. This transformation is crucial to analyze whether a divergent or slowly converging sequence may still yield meaningful asymptotic behavior.
- **Definition 11** introduces the notion of slowly oscillating sequences. This concept plays a key role in extending convergence results and establishing equivalence between summability and convergence under certain regularity conditions.
- **Theorem 12** characterizes the regularity of the Nörlund mean in terms of a condition involving the weight sequence v_n and the total weight V_n. This criterion is essential to determine under what circumstances the Nörlund mean preserves the limit of convergent sequences.
- **Theorem 13** provides a similar regularity condition for the Riesz mean. This condition ensures that the Riesz transformation does not distort the convergence behavior of the original sequence, thereby enabling its use as an analytical tool.
- **Theorem 14** connects the concepts of Riesz summability and slow oscillation, showing that if a sequence is Riesz summable and slowly oscillating, then it is convergent in the given product space. This result is vital, as it bridges summability and strong convergence in the context of uncertain random variables.

These definitions and theorems lay the groundwork for the main results of the work by allowing the application of generalized summability techniques to sequences in a complex and uncertain probabilistic setting. They support a robust mathematical framework that generalizes classical results and extends them to more realistic models involving uncertainty.

Definition 9. Let $\psi = \{\psi_n : n \geq 1\} = (\psi_n)$ be a sequence of complex uncertain random variables in the product space $(\Xi \times \Omega, \mathfrak{L} \times \mathfrak{A}, \mathfrak{M} \times \mathfrak{P})$. Throughout $v = \{v_n\}$ denotes a sequence of positive complex uncertain random variables and

$$V_n = V_n(\alpha, \beta) = \sum_{i=1}^{n} Ch\{(\alpha, \beta) \in \Delta|\{v_i(\alpha, \beta)\}\}$$

Definition 10. Let $v = \{v_n\}$ be a sequence of complex uncertain random variables in the product space $(\Xi \times \Omega, \mathfrak{L} \times \mathfrak{A}, \mathfrak{M} \times \mathfrak{P})$. The transformation defined by

$$y_n(\alpha, \beta) = \frac{\psi_1(\alpha, \beta)v_1 + \psi_2(\alpha, \beta)v_2 + ... + \psi_n(\alpha, \beta)v_n}{V_n}$$

is said to be the Riesz mean mean of the complex uncertain random sequence (ψ_n).

Definition 11. The sequence $\{\psi_n\} = \psi$ of complex uncertain random variables in the product space $(\Xi \times \Omega, \mathfrak{L} \times \mathfrak{A}, \mathfrak{M} \times \mathfrak{P})$ is called slowly oscillating if

$$\|Ch\{(\alpha, \beta) \in \Delta|\psi_m(\alpha, \beta)\} - Ch\{(\alpha, \beta) \in \Delta|\psi_n(\alpha, \beta)\}\| \to 0,$$

as $m, n \to \infty$ with $1 \leq m/n \to 1$.

Theorem 12. *The Ňorlund mean* $(\mathfrak{N}, v_n)$ *is regular if and only if* $\dfrac{Ch\{(\alpha,\beta)\in\Delta|v_n(\alpha,\beta)\}}{Ch\{(\alpha,\beta)\in\Delta|V_n(\alpha,\beta)\}}\to 0$ *as* $n\to\infty$.

Proof. Let the sequence $\{\psi_n\}$ be convergent to ψ, then there exists a constant $\mathfrak{H}>0$ such that $\|\psi_n(\alpha,\beta)\|<\mathfrak{H}$ for all $n\in\mathbb{N}$ and $(\alpha,\beta)\in\Delta$, since $\{\psi_n\}$ is convergent and for a given $\varepsilon>0$ there exists an integer n_0 such that $\|\psi_n(\alpha,\beta)-\psi(\alpha,\beta)\|<\dfrac{\varepsilon}{2n_0\mathfrak{H}}$ for $n>n_0$ and $(\alpha,\beta)\in\Delta$. Therefore, we obtain

$$\begin{aligned}
\|y_n(\alpha,\beta)-\psi(\alpha,\beta)\| \leq& \left\|\frac{v_1(\psi_n(\alpha,\beta)-\psi(\alpha,\beta))+...+v_{n_0}(\psi_{n-n_0}(\alpha,\beta)-\psi(\alpha,\beta)}{V_n(\alpha,\beta))}\right\| \\
&+\left\|\frac{v_{n_0+1}(\psi_{n_0+1}(\alpha,\beta)-\psi(\alpha,\beta))...+v_n(\psi_1(\alpha,\beta)-\psi(\alpha,\beta))}{V_n(\alpha,\beta)}\right\| \\
&\leq \frac{v_1\|\psi_n(\alpha,\beta)-\psi(\alpha,\beta)\|+...+v_{n_0}\|\psi_{n-n_0}(\alpha,\beta)-\psi(\alpha,\beta)\|}{V_n(\alpha,\beta))} \\
&+\frac{v_{n_0+1}\|\psi_{n_0+1}(\alpha,\beta)-\psi(\alpha,\beta)\|...+v_n\|\psi_1(\alpha,\beta)-\psi(\alpha,\beta)\|}{V_n(\alpha,\beta)} \\
&\leq \frac{v_n+v_{n-1}+...+v_{n-n_0}\frac{\varepsilon}{2}}{V_n(\alpha,\beta)}+\frac{v_{n-n_0+1}}{V_n(\alpha,\beta)}\|\psi_{n-n_0+1}(\alpha,\beta)-\psi(\alpha,\beta)\|+... \\
&+\frac{v_n}{V_n(\alpha,\beta)}\|\psi_1(\alpha,\beta)-\psi(\alpha,\beta)\| \\
&\leq \frac{\varepsilon}{2}+\frac{\varepsilon}{2n_0\mathfrak{H}}\mathfrak{G}n_0 \\
=\varepsilon.
\end{aligned}$$

The converse part can be obtained applying the technique for classical way. □

Theorem 13. *The Riesz mean* $(\mathfrak{R}, v_n)$ *is regular if and only if* $Ch\{(\alpha,\beta)\in\Delta|V_n(\alpha,\beta)\}\to\infty$ *as* $n\to\infty$.

Proof. Let the sequence (ψ_n) be convergent to ψ, then there exists a constant $\mathfrak{H}>0$ such that $\|\psi_n\|\mathfrak{H}$ for all $n\in\mathbb{N}$. For a given $\varepsilon>0$ there is an integer n_0 such that $\|\psi_n(\alpha,\beta)-\psi(\alpha,\beta)\|<\frac{\varepsilon}{2}$ for all $n>n_0$. Next, by hypothesis $V_n\to\infty$, as $n\to\infty$.

Let $\dfrac{v_1 2\mathfrak{H}+...+v_{n_0}2\mathfrak{H}}{V_n}<\dfrac{\varepsilon}{2}$, thus we obtain

$$\begin{aligned}
\|y_n(\alpha,\beta)-\psi\| =& \left\|\frac{v_1(\psi_1(\alpha,\beta)-\psi(\alpha,\beta))+...+v_{n_0}(\psi_{n_0}(\alpha,\beta)-\psi(\alpha,\beta))}{V_n(\alpha,\beta)}\right\| \\
&+\left\|\frac{v_{n_0+1}(\psi_{n_0+1}(\alpha,\beta)-\psi(\alpha,\beta))...+v_n(\psi_1(\alpha,\beta)-\psi(\alpha,\beta))}{V_n(\alpha,\beta)}\right\| \\
&\leq \frac{v_1+v_2+...+v_{n_0}}{V_n(\alpha,\beta)}\frac{\varepsilon}{2}+\frac{v_{n_0+1}\frac{\varepsilon}{2}+..+v_n\frac{\varepsilon}{2}}{V_n(\alpha,\beta)} \\
&\leq \frac{\varepsilon}{2\mathfrak{H}n_0}\frac{\mathfrak{H}n_0}{V_n(\alpha,\beta)}+\frac{\varepsilon}{2} \\
&=\varepsilon.
\end{aligned}$$

□

Theorem 14. *If* (ψ_n) *is* $(\mathfrak{R}, v)$ *summbale to* ψ *in* $(\Xi\times\Omega, \mathfrak{L}\times\mathfrak{A}, \mathfrak{M}\times\mathfrak{P})$ *and it is slowly oscillating. Then,* (ψ_n) *converges to* ψ *in* $(\Xi\times\Omega, \mathfrak{L}\times\mathfrak{A}, \mathfrak{M}\times\mathfrak{P})$.

Proof. Let's assume that $Ch\{(\alpha,\beta) \in \Delta|\psi(\alpha,\beta)\} = 0$. Now, consider $\lim_{n\to\infty} \|Ch\{(\alpha,\beta) \in \Delta|\psi_n(\alpha,\beta)\} - 0\| > 0$. Thus, there exists $\varpi > 0$ and a subsequence $\{\psi_{n_k}\}$ of $\{\psi_n\}$ with $\|Ch\{(\alpha,\beta) \in \Delta|\psi_{n_k}(\alpha,\beta)\} - Ch\{(\alpha,\beta) \in \Delta|\psi(\alpha,\beta)\}\| \geq \varpi$ for all $k \in \mathbb{N}$. Now, let $\{\psi_n\}$ be slowly oscillating, then $\{\psi_{n_k}\}$ as a subsequence of $\{\psi_n\}$ is also slowly oscillating. Then, for a give δ, there exists $u_0 \in \mathbb{N}$ such that $u_0 \leq n \leq m < (1+\delta)n$, therefore $\|Ch\{(\alpha,\beta) \in \Delta|\psi_m(\alpha,\beta)\} - Ch\{(\alpha,\beta) \in \Delta|\psi_n(\alpha,\beta)\}\| < \frac{\varpi}{2}$. Since $\{\psi_n\}$ is summable to 0, let $\sigma_n(\alpha,\beta) = \frac{1}{V_n}\sum_{i=1}^{n} v_i\psi_n(\alpha,\beta)$ be convergent to 0 in $(\Xi \times \Omega, \mathfrak{L} \times \mathfrak{A}, \mathfrak{M} \times \mathfrak{P})$. Now, for $Ch\{(\alpha,\beta) \in \Delta|m_i\} \geq Ch\{(\alpha,\beta) \in \Delta|n_i\}$, we obtain

$$\begin{aligned}
\sigma_{m_i} - \frac{V_{n_i}}{V_{m_i}}\sigma_{n_i} &= \frac{1}{V_{m_i}}\sum_{j=1}^{m_i} v_j\psi_j(\alpha,\beta) - \frac{V_{n_j}}{V_{m_j}}\sum_{j=1}^{m_i} v_j\psi_j(\alpha,\beta)\\
&= \frac{1}{V_{m_i}}\sum_{j=n_i+1} v_j\psi_j(\alpha,\beta).
\end{aligned}$$

So, for all $n_i > u - 1$ and $n_i < m < m_i = [(1+\delta)n_i]$, where $[\cdot]$ denotes the integral part of $\cdot$. Then, we get

$$\begin{aligned}
&\|Ch\{(\alpha,\beta) \in \Delta|0\} - Ch\{(\alpha,\beta) \in \Delta|\psi_m(\alpha,\beta)\|\\
&\geq \|Ch\{(\alpha,\beta) \in \Delta|0\} - Ch\{(\alpha,\beta) \in \Delta|\psi_n(\alpha,\beta)\|\\
&- \|Ch\{(\alpha,\beta) \in \Delta|\psi_n(\alpha,\beta)\} - Ch\{(\alpha,\beta) \in \Delta|\psi_m(\alpha,\beta)\|\\
&\geq \varpi - \frac{\varpi}{2}.
\end{aligned}$$

Besides, we obtain

$$\begin{aligned}
&\|Ch\{(\alpha,\beta) \in \Delta|\sigma_{m_i}(\alpha,\beta)\} - Ch\{(\alpha,\beta) \in \Delta|\sigma_{n_i}(\alpha,\beta)\|\\
&+ \|Ch\{(\alpha,\beta) \in \Delta|\sigma_{n_i}(\alpha,\beta)\} - \frac{V_{n_i}}{V_{m_i}}Ch\{(\alpha,\beta) \in \Delta|\sigma_{n_i}(\alpha,\beta)\|\\
&\geq \|Ch\{(\alpha,\beta) \in \Delta|\sigma_{m_i}(\alpha,\beta)\} - \frac{V_{n_i}}{V_{m_i}}Ch\{(\alpha,\beta) \in \Delta|\sigma_{n_i}(\alpha,\beta)\|\\
&\geq \left\|\frac{1}{V_{m_i}}\sum_{j=n_i+1}^{m_i} v_j(\alpha,\beta)\psi_j(\alpha,\beta) - 0\right\|\\
&\geq \left\|\frac{v_{m_i} - v_{n_i}}{V_{m_i}}\psi_{n_i}(\alpha,\beta) - 0\right\| - \left\|\sum_{j=n_i+1}^{m_i}\frac{v_j\psi_j - v_{n_i}\psi_{n_i}}{V_{m_i}} - 0\right\|\\
&\geq \frac{v_{m_i} - v_{n_i}}{V_{m_i}}\|\psi_{n_i}(\alpha,\beta) - 0\| - \frac{v_{m_i} - v_{n_i}}{V_{m_i}}\|\psi_j(\alpha,\beta) - \psi_{n_i}(\alpha,\beta)\|\\
&\geq \frac{v_{m_i} - v_{n_i}}{V_{m_i}}\varpi - \frac{v_{m_i} - v_{n_i}}{V_{m_i}}\frac{\varpi}{2}\\
&\geq \frac{v_{m_i} - v_{n_i}}{V_{m_i}}\left(\varpi - \frac{\varpi}{2}\right)\\
&\geq \frac{v_{m_i} - v_{n_i}}{V_{m_i}}\frac{\delta}{1+\delta}\\
&\geq 0.
\end{aligned}$$

Hence, for all $m_i > n_i > \mathfrak{N}$, we get

$$\begin{aligned}
&\|Ch\{(\alpha,\beta)\in\Delta|\sigma_{m_i}(\alpha,\beta)\} - Ch\{(\alpha,\beta)\in\Delta|\sigma_{n_i}(\alpha,\beta)\|\\
&+\|Ch\{(\alpha,\beta)\in\Delta|\sigma_{n_i}(\alpha,\beta)\} - \frac{V_{n_i}}{V_{m_i}}Ch\{(\alpha,\beta)\in\Delta|\sigma_{n_i}(\alpha,\beta)\|\\
&\geq \|Ch\{(\alpha,\beta)\in\Delta|\sigma_{m_i}(\alpha,\beta)\} - \frac{V_{n_i}}{V_{m_i}}Ch\{(\alpha,\beta)\in\Delta|\sigma_{n_i}(\alpha,\beta)\|\\
&\geq \frac{\varpi}{2}\frac{\delta}{1+\delta}.
\end{aligned}$$

Therefore,

$$0 = \lim \|Ch\{(\alpha,\beta)\in\Delta|\sigma_{m_i}(\alpha,\beta)\} - \frac{V_{n_i}}{V_{m_i}}Ch\{(\alpha,\beta)\in\Delta|\sigma_{n_i}(\alpha,\beta)\| > \frac{\varpi}{2}\frac{\delta}{1+\delta}$$

which is a contradiction of the fact that $\{\psi_i(\alpha,\beta)\}$ converges in $(\Xi\times\Omega, \mathfrak{L}\times\mathfrak{A}, \mathfrak{M}\times\mathfrak{P})$. □

4. Conclusion

We have procured the idea of Ñorlund-Riesz mean of complex uncertain random variables. Besides, we have established some properties. The results presented in this paper can be used for data analysis.

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Conflicts of Interest

The authors declare no conflict of interest.

Data availability statement

This manuscript has no associated data.

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